The Mathematical Knowledge Needed for Teaching: An Inquiry into the Knowledge of Preservice and Practicing Teachers

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DeAnn Huinker & Melissa Hedges
Milwaukee Mathematics Partnership
University of Wisconsin-Milwaukee

huinker@uwm.edu, mhedges@uwm.edu
Web site: www.mmp.uwm.edu

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What distinguishes mathematical knowledge from the specialized knowledge needed for teaching mathematics?

What mathematics matters for teaching?
Is it different from mathematics knowledge as known and used by mathematicians, engineers, biologists, candy sellers? If so, how can we characterize what makes it different?

What of that mathematics is “content,” and what is more properly called by other names such as “practices,” “sensibilities,” “orientations”?

What is the difference between “pure” content knowledge and knowledge used in pedagogical situations?
Mathematical Knowledge for Teaching (MKT)

- **Common mathematical knowledge**
  - Number halfway between 1.1 and 1.2

- **Specialized knowledge for teaching mathematics**
  - Representing mathematical ideas and operations
  - Providing explanations for mathematical ideas and procedures
  - Appraising unusual student methods, claims, or solutions
Solve:

\[ 25 \times 35 = ? \]
Which of these students is using a method that could be used to multiply any two whole numbers?

<table>
<thead>
<tr>
<th>Student A</th>
<th>Student B</th>
<th>Student C</th>
</tr>
</thead>
<tbody>
<tr>
<td>35 \times 25</td>
<td>35 \times 25</td>
<td>35 \times 25</td>
</tr>
<tr>
<td>125</td>
<td>175</td>
<td>25</td>
</tr>
<tr>
<td>+75</td>
<td>+700</td>
<td>150</td>
</tr>
<tr>
<td><strong>875</strong></td>
<td><strong>875</strong></td>
<td><strong>875</strong></td>
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**Appraising Unusual Student Solutions**
Reflect and Discuss

- What did you do to solve the original problem of 25 x 35?
- What did you do to solve the task concerning the varied student solutions?
- What was different in the way you thought mathematically about each task, and what you needed to know?
Mrs. Johnson thinks it is important to vary the whole when she teaches fractions. For example, she might use five dollars to be the whole, or ten students, or a single rectangle. On one particular day, she uses as the whole a picture of two pizzas. What fraction of the two pizzas is she illustrating below? (Mark ONE answer.)

a) 5/4
b) 5/3
c) 5/8
d) 1/4
Ms. Harris was working with her class on divisibility rules. She told her class that a number is divisible by 4 if and only if the last two digits of the number are divisible by 4. One of her students asked her why the rule for 4 worked. She asked the other students if they could come up with a reason, and several possible reasons were proposed. Which of the following statements comes closest to explaining the reason for the divisibility rule for 4?

a) Four is an even number, and odd numbers are not divisible by even numbers.

b) The number 100 is divisible by 4 (and also 1000, 10,000, etc.).

c) Every other even number is divisible by 4, for example, 24 and 28 but not 26.

d) It only works when the sum of the last two digits is an even number.
Common vs. Specialized Mathematical Knowledge

• Individuals can be strong in common mathematical knowledge, but not specialized mathematical knowledge for teaching, and vice versa.

• Suggests there is professional knowledge for teaching mathematics.
Items

• Measure mathematics that teachers *use* in teaching, not just *what* they teach.
• Orient the items around problems or tasks that all teachers might face in teaching mathematics
Context for Items: Tasks of Teaching Mathematics

- **Representations**: Make decisions on representations of mathematical ideas and operations
- **Explanations**: Provide explanations for mathematical ideas
- **Evaluate Student Thinking**: Appraise unusual student methods, claims, justifications, and errors
Study of Teacher Growth in a Professional Development Program for Teacher Leaders in Mathematics
Setting

- Content Strand: Number and Operations
  - Focus: Rational numbers
- Pretest: October
- Met for about 20 hours on content; monthly sessions over the school year
- Posttest: June (same instrument)
Teacher Growth in Mathematical Knowledge for Teaching (MKT)

• Math Teacher Leaders (MTL) gained roughly \( \frac{1}{2} \) standard deviation.

• Statistically significant improvement in the MKT ability of Math Teacher Leaders \( (t_{77} = 4.14, p < 0.001) \)
For More Information

• Learning Mathematics for Teaching – http://sitemaker.umich.edu/lmt