A. Mathematical Processes and C. Geometry
(From the WKCE-CRT Mathematics Assessment Framework, Beginning of Grade 10)

A. Mathematical Processes

• Use reasoning and logic to: perceive patterns, identify relationships, formulate questions, pose problems, make conjectures, justify strategies and test reasonableness of results.
• Communicate mathematical ideas and reasoning using the vocabulary of mathematics in a variety of ways, e.g., using words, numbers, symbols, pictures, charts, tables, diagrams, graphs, and models.
• Connect mathematics to the real world as well as within mathematics.
• Create and use representations to organize, record, and communicate mathematical ideas.
• Solve and analyze routine and non-routine problems.

C. Geometry *

Subskill: C. a: Describing Figures
• Identify, describe and analyze properties of figures, relationships among figures and relationships among their parts (e.g., parallel, perpendicular and congruent sides, various types of angles and triangles, complementary and supplementary angles, sum of angles in a triangle).
• Model problems using the Pythagorean Theorem and *right triangle trigonometry.

Subskill: C. b: Spatial Relationships and Transformations
• Use proportional reasoning to solve congruence and similarity problems (e.g., scale drawings and similar geometric figures).
• Use transformations and symmetry to solve problems
• Visualize 3-dimensional figures in problem-solving situations.

Subskill: C. c: Coordinate Systems
• Use the two-dimensional rectangular coordinate system to describe and characterize properties of geometric figures. Identify and apply symmetry about an axis.

* Slightly modified to reflect “Sharing the 10th Grade Descriptors”
Examining the 10th Gr. Descriptors: What’s “New” for 8th Graders?

The 8th Grade Classroom Assessments Based on State Standards were created to reflect the 10th grade state descriptors and the MPS learning targets for 8th grade. The 10th grade state descriptors should be used by both 8th and 9th grade math teachers. See your MTL for the document, “Sharing the 10th Grade Descriptors”, or visit the Portal (math page, inside the teacher community), for the delineated shared responsibilities.

The 10th grade state descriptors contain both previously introduced and new concepts. It is the intent of the 8th - 9th Grades Math Assessment Pilot, after considerable feedback from Kevin McLeod, UWM Mathematics Professor and Co-Investigator, MMP, to emphasize the new concepts. Alerting teachers to what is new for 8th graders should have classroom implications.

The 8th grade math CABS are therefore split into two categories. First, a series of “Power CABS” which can be used to assess student understanding of concepts not found in earlier grade level state descriptors. Second, a series of CABS not denoted with “Power” which can be used to assess student understanding of concepts found in both earlier state descriptors and in the 10th grade descriptors.

The chart below identifies the descriptors that are new to 8th grade as determined by the 8th - 9th Grades Math Assessment Pilot, 2006-2007.

<table>
<thead>
<tr>
<th>Objective And Subskill</th>
<th>10th Gr. Descriptor Piece that is “New”</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>C.a</td>
<td>“Identify, describe and analyze properties of figures, relationships among figures and relationships among their parts (e.g., parallel, perpendicular and congruent sides, various types of angles and triangles, complementary and supplementary angles, sum of angles in a triangle)”</td>
<td>This whole descriptor is new for 8th!</td>
</tr>
<tr>
<td>C.a</td>
<td>“Model problems using the Pythagorean Theorem”</td>
<td>The Pythagorean Theorem is new to 8th grade, both in Geometry and in Measurement.</td>
</tr>
<tr>
<td>C.b</td>
<td>“Use proportional …”</td>
<td>“Fundamentally” not new?</td>
</tr>
<tr>
<td>C.b</td>
<td>“Use transformations and symmetry to solve problems.”</td>
<td>A bit more “open” than previous grades.</td>
</tr>
<tr>
<td>C.b</td>
<td>“Visualize 3-dimensional figures in problem-solving situations.”</td>
<td>Use more general shapes?</td>
</tr>
<tr>
<td>C.c</td>
<td>“Use the two-dimensional rectangular coordinate system to describe and characterize properties of geometric figures. Identify and apply symmetry about an axis.”</td>
<td>This whole descriptor is new for 8th!</td>
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</table>
MPS Learning Target: Geometry

MPS Learning Target #2 Analyze relationships among figures and their parts, including the Pythagorean Theorem, using properties of plane and coordinate geometry, and use proportional reasoning, transformations, and spatial visualization in problem-solving situations.

Wisconsin Assessment Framework for Mathematics

Objective: C. Geometry
Subskill: Describing Figures
Descriptor:
• Identify, describe and analyze properties of figures, relationships among figures and relationships among their parts (e.g., parallel, perpendicular and congruent sides, various types of angles and triangles, complementary and supplementary angles, sum of angles in a triangle).

Objective: A. Mathematical Processes
Descriptors:
• Use reasoning and logic to: perceive patterns, identify relationships, formulate questions, pose problems, make conjectures, justify strategies and test reasonableness of results.
• Communicate mathematical ideas and reasoning using the vocabulary of mathematics in a variety of ways, e.g., using words, numbers, symbols, pictures, charts, tables, diagrams, graphs, and models.
• Solve and analyze routine and non-routine problems.

A certain 4-sided figure has the following properties:
• Only one pair of opposite sides are parallel.
• Only one pair of opposite sides are equal in length.
• The parallel sides are not equal in length.

Which of the following must be true about the sides that are equal in length?

A. They are perpendicular to each other.
B. They are each perpendicular to an adjacent side.
C. They are equal in length to one of the other two sides.
D. They are not equal in length to either of the other two sides.
E. They are not parallel.

Sketch the figure here.

Adapted from the U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP)

Developed by the Milwaukee Mathematics Partnership (MMP) with support by the National Science Foundation under Grant No. 0314898.
Mathematics Grade 8
Classroom Assessment Based on Standards

Power CABS Identifier: “Parallelogram WXYZ”

MPS Learning Target: Geometry

MPS Learning Target #2 Analyze relationships among figures and their parts, including the Pythagorean Theorem, using properties of plane and coordinate geometry, and use proportional reasoning, transformations, and spatial visualization in problem-solving situations.

Wisconsin Assessment Framework for Mathematics

Objective: C. Geometry
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Descriptor:
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Objective: A. Mathematical Processes
Descriptor:
• Use reasoning and logic to: perceive patterns, identify relationships, formulate questions, pose problems, make conjectures, justify strategies and test reasonableness of results.
• Solve and analyze routine and non-routine problems.

In the figure above, WXYZ is a parallelogram. Which of the following is NOT necessarily true?

A. Side WX is parallel to side ZY.
B. Side XY is parallel to side WZ.
C. The measures of angles $W$ and $Y$ are equal.
D. The lengths of sides WX and ZY are equal.
E. The lengths of sides WX and XY are equal.

U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP)

Developed by the Milwaukee Mathematics Partnership (MMP) with support by the National Science Foundation under Grant No. 0314898.
Alan says that if a figure has four sides, it must be a rectangle. Gina does not agree.

Who do you think is correct? ________________

Explain your thinking.
Draw a picture to support your answer.
Mathematics Grade 8  
Classroom Assessment Based on Standards  
CABS Identifier: “Describing Figures”

<table>
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<th>MPS Learning Target: Geometry</th>
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<td><strong>MPS Learning Target #2</strong> Analyze relationships among figures and their parts, including the Pythagorean Theorem, using properties of plane and coordinate geometry, and use proportional reasoning, transformations, and spatial visualization in problem-solving situations.</td>
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Fully describe the drawing above using as many geometric terms as you can.
MPS Learning Target: Geometry

MPS Learning Target #2: Analyze relationships among figures and their parts, including the Pythagorean Theorem, using properties of plane and coordinate geometry, and use proportional reasoning, transformations, and spatial visualization in problem-solving situations.

Wisconsin Assessment Framework for Mathematics

Objective: C. Geometry
Subskill: Describing Figures
Descriptor:
• Identify, describe and analyze properties of figures, relationships among figures and relationships among their parts (e.g., parallel, perpendicular and congruent sides, various types of angles and triangles, complementary and supplementary angles, sum of angles in a triangle).

Objective: A. Mathematical Processes
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• Communicate mathematical ideas and reasoning using the vocabulary of mathematics in a variety of ways, e.g., using words, numbers, symbols, pictures, charts, tables, diagrams, graphs, and models.
• Solve and analyze routine and non-routine problems.

If line $l$ is parallel to line $m$, and line $l$ is perpendicular to line $p$, what conclusions can you draw about lines $l$, $m$, and $p$? Make a sketch to represent the given information.
Scott said, “The larger the radius, the greater the circumference.” Do you agree with him?

Explain using words, pictures, and/or numbers.
### Mathematics Grade 8

**Classroom Assessment Based on Standards**

*Power* CABS Identifier: “Line Segment”

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| Objective: A. Mathematical Processes          |
| Descriptors:                                 |
| • Use reasoning and logic to: perceive patterns, identify relationships, formulate questions, pose problems, make conjectures, justify strategies and test reasonableness of results. |
| • Communicate mathematical ideas and reasoning using the vocabulary of mathematics in a variety of ways, e.g., using words, numbers, symbols, pictures, charts, tables, diagrams, graphs, and models. |
| • Create and use representations to organize, record, and communicate mathematical ideas. |
| • Solve and analyze routine and non-routine problems. |

A. The endpoints of a line segment are the points with coordinates (2, 1) and (8, 9). What are the coordinates of the midpoint of this line segment?

A. (4, 3)  
B. (3, 4)  
C. (5, 5)  
D. (2, 3½)  
E. (10, 10)  

Find the length of the line segment. Show all your work.
The paper tube in the figure above is to be cut along the dotted line and opened up. The height of the tube is 10cm. The diameter is 4cm. What will be the shape and dimensions of the flattened piece of paper?

Draw the flat pattern for the paper tube. Label all of the dimensions of the flattened piece of paper.
Amanda put 8 small cubes together to make a bigger cube. She is going to paint all the outside faces of each of the smaller cubes a different color.

How many different colors will she need? Answer______________________

How many sides of all of the smaller cubes will not be painted? Answer______________________

Justify your answers using words, pictures, and/or numbers.
### MPS Learning Target: Geometry

**MPS Learning Target #2** Analyze relationships among figures and their parts, including the Pythagorean Theorem, using properties of plane and coordinate geometry, and use proportional reasoning, transformations, and spatial visualization in problem-solving situations.

### Wisconsin Assessment Framework for Mathematics

**Objective: C. Geometry**  
Subskill: Describing Figures  
**Descriptor:**  
- Model problems using the Pythagorean Theorem and right triangle trigonometry.

**Objective: A. Mathematical Processes**  
**Descriptors:**  
- Use reasoning and logic to: perceive patterns, identify relationships, formulate questions, pose problems, make conjectures, justify strategies and test reasonableness of results.  
- Communicate mathematical ideas and reasoning using the vocabulary of mathematics in a variety of ways, e.g., using words, numbers, symbols, pictures, charts, tables, diagrams, graphs, and models.  
- Connect mathematics to the real world as well as within mathematics.  
- Create and use representations to organize, record, and communicate mathematical ideas.  
- Solve and analyze routine and non-routine problems.

The Johnson family went shopping for a new television for the Super Bowl. The salesperson told them that the length of the diagonal on the rectangular screen gives the television size.

Give one possible length, width, and diagonal:

<table>
<thead>
<tr>
<th>Length</th>
<th>Width</th>
<th>Diagonal</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Justify your answer using words, pictures, and/or numbers.
Mathematics Grade 8
Classroom Assessment Based on Standards

Power CABS Identifier: “Sam’s Solution”

MPS Learning Target: Geometry

MPS Learning Target #2 Analyze relationships among figures and their parts, including the Pythagorean Theorem, using properties of plane and coordinate geometry, and use proportional reasoning, transformations, and spatial visualization in problem-solving situations.

Wisconsin Assessment Framework for Mathematics

Objective: C. Geometry
Subskill: Describing Figures
Descriptor:
• Model problems using the Pythagorean Theorem and *right triangle trigonometry.

Objective: A. Mathematical Processes
Descriptors:
• Use reasoning and logic to: perceive patterns, identify relationships, formulate questions, pose problems, make conjectures, justify strategies and test reasonableness of results.
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• Solve and analyze routine and non-routine problems.

On a recent exam, Sam solved the problem as shown below. The teacher marked the problem wrong.

Sam’s Work

\[ a^2 + b^2 = c^2 \]
\[ 6^2 + 10^2 = c^2 \]
\[ 36 + 100 = c^2 \]
\[ 136 \text{ cm} = c \]

Explain to Sam what the correct answer is, and how he could fix his mistake.
MPS Learning Target: Geometry

MPS Learning Target #2 Analyze relationships among figures and their parts, including the Pythagorean Theorem, using properties of plane and coordinate geometry, and use proportional reasoning, transformations, and spatial visualization in problem-solving situations.

Wisconsin Assessment Framework for Mathematics

Objective: C. Geometry
Subskill: Spatial Relationships and Transformations
Descriptor:
• Use proportional reasoning to solve congruence and similarity problems (e.g., scale drawings and similar geometric figures).

Objective: A. Mathematical Processes
Descriptors:
• Use reasoning and logic to: perceive patterns, identify relationships, formulate questions, pose problems, make conjectures, justify strategies and test reasonableness of results.
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• Solve and analyze routine and non-routine problems.

Which of the following triangles is similar to the triangle shown above? Circle all that apply.
MPS Learning Target: Geometry

MPS Learning Target #2: Analyze relationships among figures and their parts, including the Pythagorean Theorem, using properties of plane and coordinate geometry, and use proportional reasoning, transformations, and spatial visualization in problem-solving situations.

Wisconsin Assessment Framework for Mathematics

Objective: C. Geometry
Subskill: Spatial Relationships and Transformations
Descriptor:
• Visualize 3-dimensional figures in problem-solving situations.

Objective: A. Mathematical Processes
Descriptors:
• Use reasoning and logic to: perceive patterns, identify relationships, formulate questions, pose problems, make conjectures, justify strategies and test reasonableness of results.
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• Solve and analyze routine and non-routine problems.

Sadie drew the shape below from three different perspectives. She made a few mistakes.

Sadie’s three drawings:

Front
Top
Right
Circle one of Sadie’s mistakes and explain or demonstrate her misconception.
MPS Learning Target: Geometry

MPS Learning Target #2: Analyze relationships among figures and their parts, including the Pythagorean Theorem, using properties of plane and coordinate geometry, and use proportional reasoning, transformations, and spatial visualization in problem-solving situations.

Wisconsin Assessment Framework for Mathematics

Objective: C. Geometry
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• Solve and analyze routine and non-routine problems.

Analyze the drawing below.

Identify the appropriate relationship between the following pairs of angles.

Complementary  Supplementary  Neither

\(\angle 1 \text{ and } \angle 2\)  \(\square\)  \(\square\)  \(\square\)

\(\angle 2 \text{ and } \angle 3\)  \(\square\)  \(\square\)  \(\square\)
<table>
<thead>
<tr>
<th>Angle Pairs</th>
<th>Complementary</th>
<th>Supplementary</th>
<th>Neither</th>
</tr>
</thead>
<tbody>
<tr>
<td>∠3 and ∠4</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
</tr>
<tr>
<td>∠4 and ∠5</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
</tr>
<tr>
<td>∠5 and ∠1</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
</tr>
</tbody>
</table>

Referring to the figure, describe how you know that two angles are complementary.

Referring to the figure, describe how you know that two angles are supplementary.
Mathematics Grade 8
Classroom Assessment Based on Standards

Power CABS Identifier: “Making Conjectures”

MPS Learning Target: Geometry

MPS Learning Target #2: Analyze relationships among figures and their parts, including the Pythagorean Theorem, using properties of plane and coordinate geometry, and use proportional reasoning, transformations, and spatial visualization in problem-solving situations.

Wisconsin Assessment Framework for Mathematics

Objective: C. Geometry
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• Identify, describe and analyze properties of figures, relationships among figures and relationships among their parts (e.g., parallel, perpendicular and congruent sides, various types of angles and triangles, complementary and supplementary angles, sum of angles in a triangle)

Objective: A. Mathematical Processes
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• Use reasoning and logic to perceive patterns, identify relationships, formulate questions, pose problems, make conjectures, justify strategies, and test reasonableness of results
• Communicate mathematical ideas and reasoning using the vocabulary of mathematics in a variety of ways (e.g. using words, numbers, symbols, pictures, charts, tables, diagrams, graphs, and models).
• Connect mathematics to the real world as well as within mathematics.
• Create and use representations to organize, record, and communicate mathematical ideas.
• Solve and analyze routine and non-routine problems.

Mrs. King’s students observed Pi Day on 3-14 by carefully taking measurements of circular objects (lids, DVDs, the classroom clock, pies, etc.) and discussing their discoveries. Below is their table of data rounded to the nearest centimeter. Some of the numbers were erased.

Our Class’ Circle Data

<table>
<thead>
<tr>
<th>Diameter (cm)</th>
<th>Radius (cm)</th>
<th>Circumference (cm)</th>
<th>?</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>5</td>
<td>31</td>
<td>79</td>
</tr>
<tr>
<td>6</td>
<td></td>
<td>20</td>
<td>28</td>
</tr>
<tr>
<td></td>
<td>9</td>
<td>55</td>
<td>254</td>
</tr>
<tr>
<td>15</td>
<td>8</td>
<td>46</td>
<td>201</td>
</tr>
<tr>
<td>58</td>
<td>29</td>
<td>2,641</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>60</td>
<td>283</td>
</tr>
</tbody>
</table>
Analyze the data in the first three columns. Make a conjecture about the relationship between the radius and circumference or the diameter and circumference.

What is the heading for the fourth column? Answer: ___________________

Justify your choice for the heading.

Fill in the missing numbers in the table, and show all of your work below.
**MPS Learning Target: Geometry**

**MPS Learning Target #2** Analyze relationships among figures and their parts, including the Pythagorean Theorem, using properties of plane and coordinate geometry, and use proportional reasoning, transformations, and spatial visualization in problem-solving situations.

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**Wisconsin Assessment Framework for Mathematics**

**Objective: C. Geometry**

**Subskill: Spatial Relationships and Transformations**

**Descriptor:**
- Use transformations and symmetry to solve problems.

**Objective: A. Mathematical Processes**

**Descriptors:**
- Communicate mathematical ideas and reasoning using the vocabulary of mathematics in a variety of ways, e.g., using words, numbers, symbols, pictures, charts, tables, diagrams, graphs, and models.
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The Acme Ambulance Company is designing a logo for the front of their ambulances. The logo is to include the company’s name in upper case letters: ACME. What has to be painted on the front of the ambulance so that drivers will see the word ACME in their rear-view mirrors?

**Answer:** ____________________________

Using appropriate math vocabulary, explain how you know your answer is correct.

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Developed by the *Milwaukee Mathematics Partnership* (MMP) with support by the National Science Foundation under Grant No. 0314898.