Developing Geometric Reasoning Part 1

Math Teacher Leader Seminar
December 7, 9, & 14, 2006

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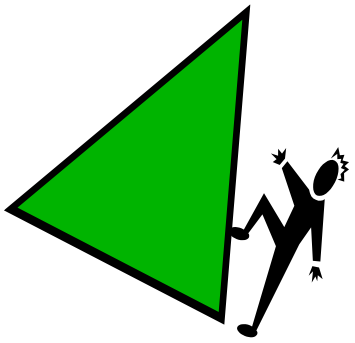

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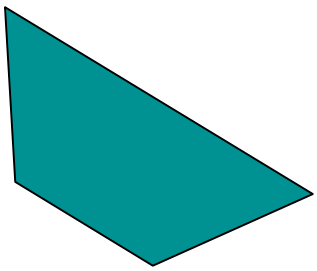
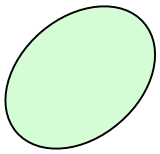
Session Goals

- Examine how students' understanding of geometry develops through defined stages, the van Hiele Model.
- Consider implications for instruction to move students along in developing their geometric reasoning.
- Investigate the Wisconsin sub-skill area of "Describing Figures."





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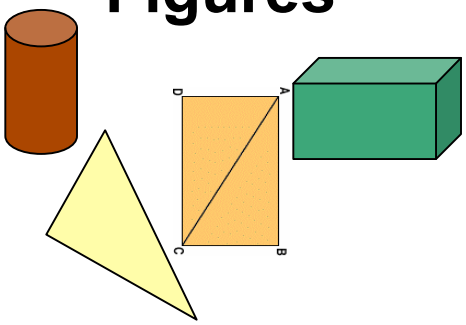
Geometry

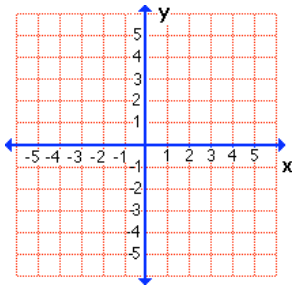
Wisconsin Mathematics Standard

Describing Figures

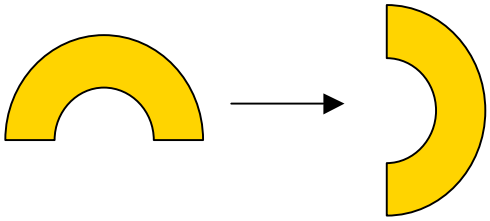


Sub-skill Areas

Coordinate Systems



Spatial Relationships and Transformations



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What is a Triangle?

On the 3-by-5 card

Write a definition of a triangle.

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Considering Triangle Characteristics

Turn the card over.

→ Draw a triangle.

→ Draw another triangle that is different from the triangle you just drew.

→ Draw a third triangle that, again, is different than the previous triangles.

→ Finally, draw a fourth triangle that is different than any of the previous triangles.

What is different about the triangles? What is similar?

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How do students progress in developing geometric reasoning?

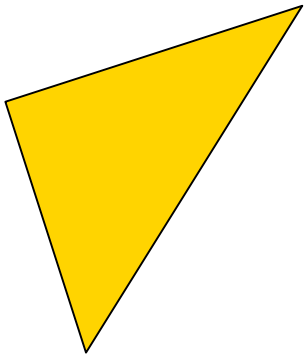
Read: page 311: "Levels of Geometric Thinking" of the article *Begin with Play* by Pierre van Hiele.

Focus:

- How would you recognize each of these levels of thinking in your students' work?

- Considering the three levels discussed in this article, where would you place the majority of the lessons that you teach?

van Hiele Levels of Geometric Reasoning



Level 0: Visualization

Recognize figures as total entities, but do not recognize properties.

Level 1: Analysis (Description)

Identify properties of figures and see figures as a class of shapes.

Level 2: Informal Deduction

Formulate generalizations about relationships among properties of shapes; Develop informal explanations.

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Level 3: Deduction

Understand the significance of deduction as a way of establishing geometric theory within an axiom system. See interrelationship and role of undefined terms, axioms, definitions, theorems and formal proof. See possibility of developing a proof in more than one way.

Level 4: Rigor

Compare different axiom systems (e.g., non-Euclidean geometry). Geometry is seen in the abstract with a high degree of rigor, even without concrete examples.

"I believe that development is more dependent on instruction than on age or biological maturation and that types of instructional experiences can foster, or impede, development."

Pierre M. van Hiele

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Tricky Triangles

Envelope. . . has a selection of shapes.

Goal… Sort the shapes into 2 groups.

"Triangles" and "Not Triangles"

Process…

- Pull out a card (without looking).
- Show it to the table group.
- Explain why it is or isn't a triangle.

Pass the Envelope…
to the next person and repeat the process until all the shapes are sorted.

Group Discussion…
What defines a triangle?

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Reviewing Student Work

Assign Roles

Facilitator: Give all a voice.
Recorder: Take notes on record sheet.

Directions

Distribute one or two work samples to each group member.

Review what "***student understands***" and for "***student misconceptions***" (silently).

Taking turns, present your observations.

Table Group Discussion

What are some instructional implications you will take back to your school?

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Tricky Triangle Student Work

Student	Student Understands	Student Misconceptions	van Hiele Level
A			
B			
C			
D			
E			
F			

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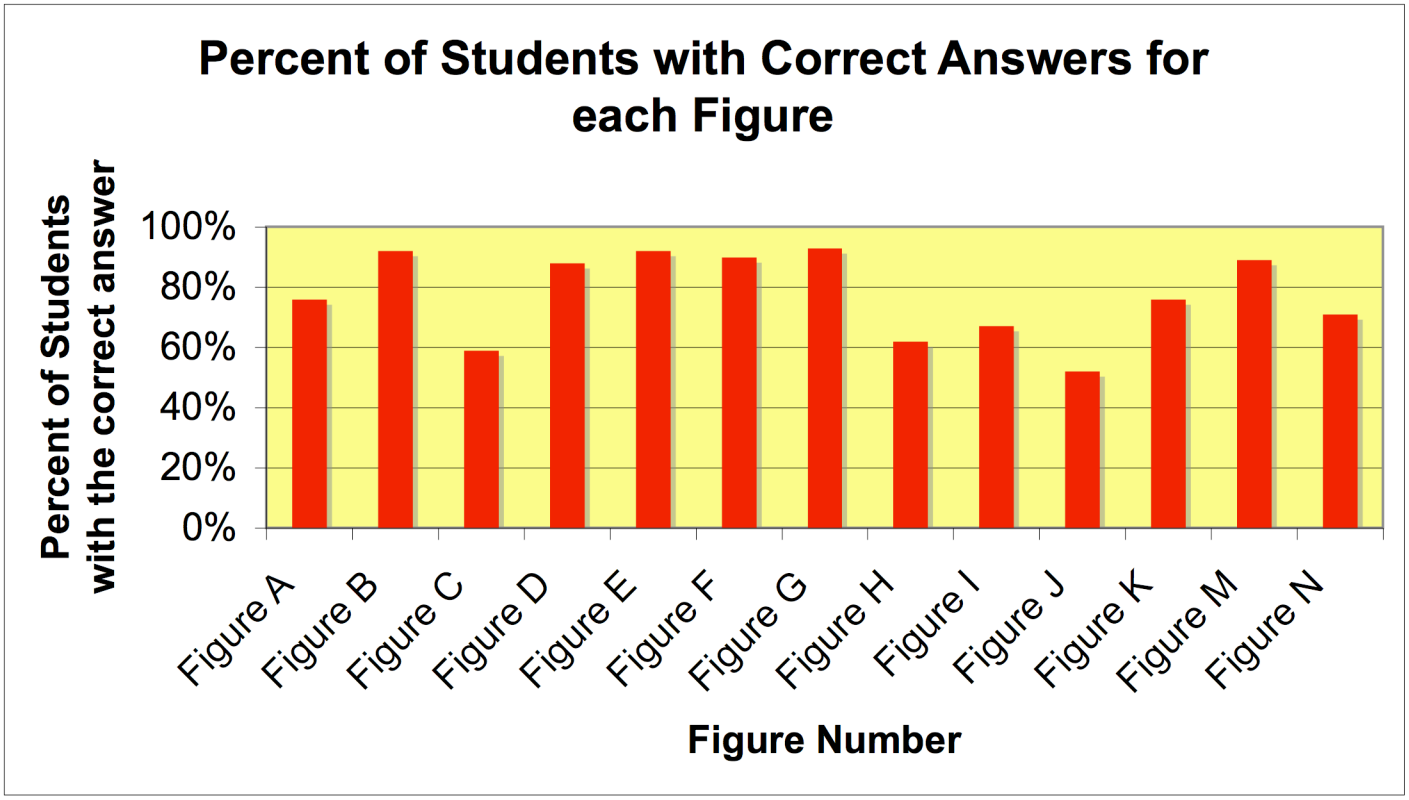
(continued)

Student	Student Understands	Student Misconceptions	van Hiele Level
G			
H			
I			
J			
K			

Tricky Triangles
Percent of Correct Responses for MPS Students in Grades 3–11



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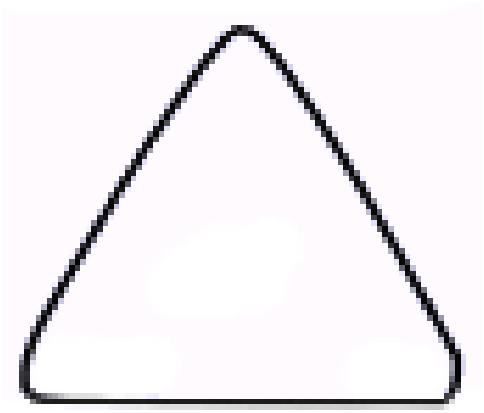
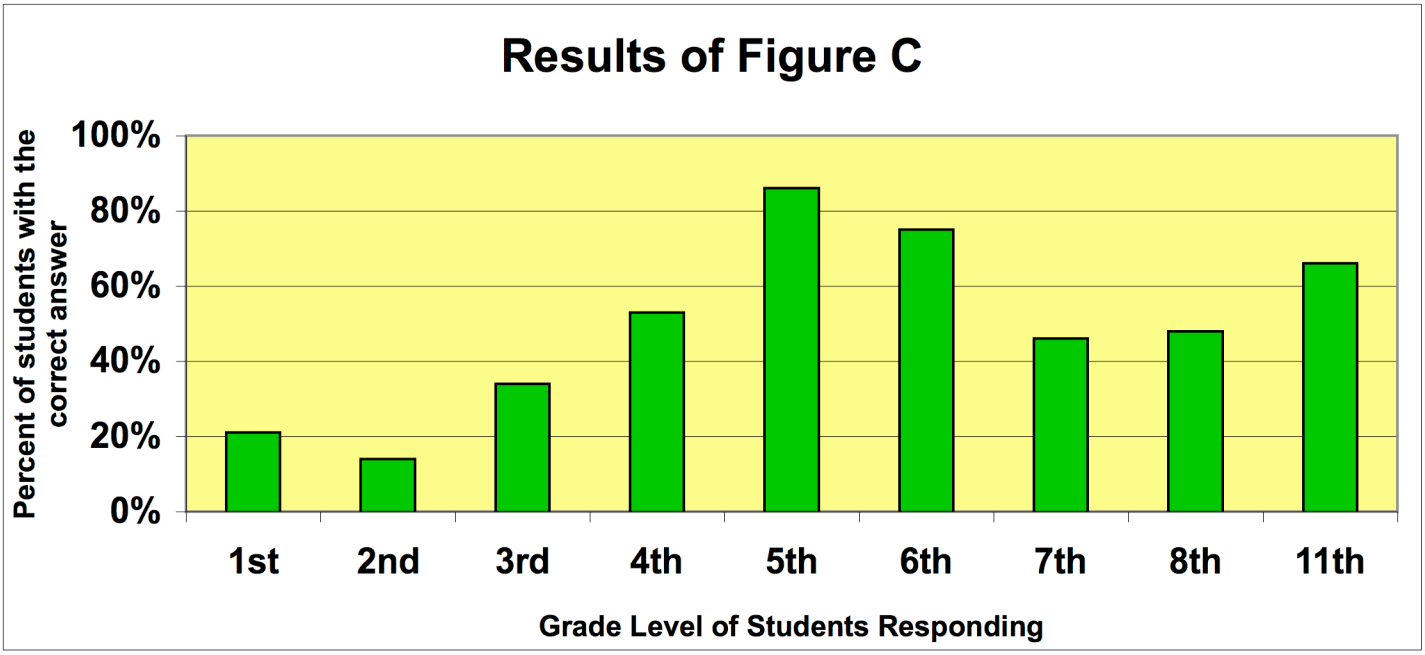
Results for Figure C

Percent Correct by Grade





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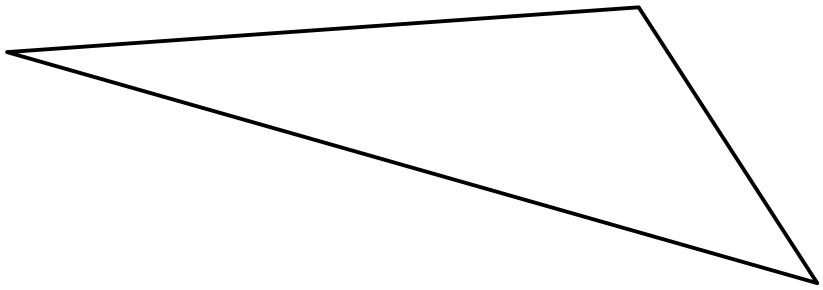
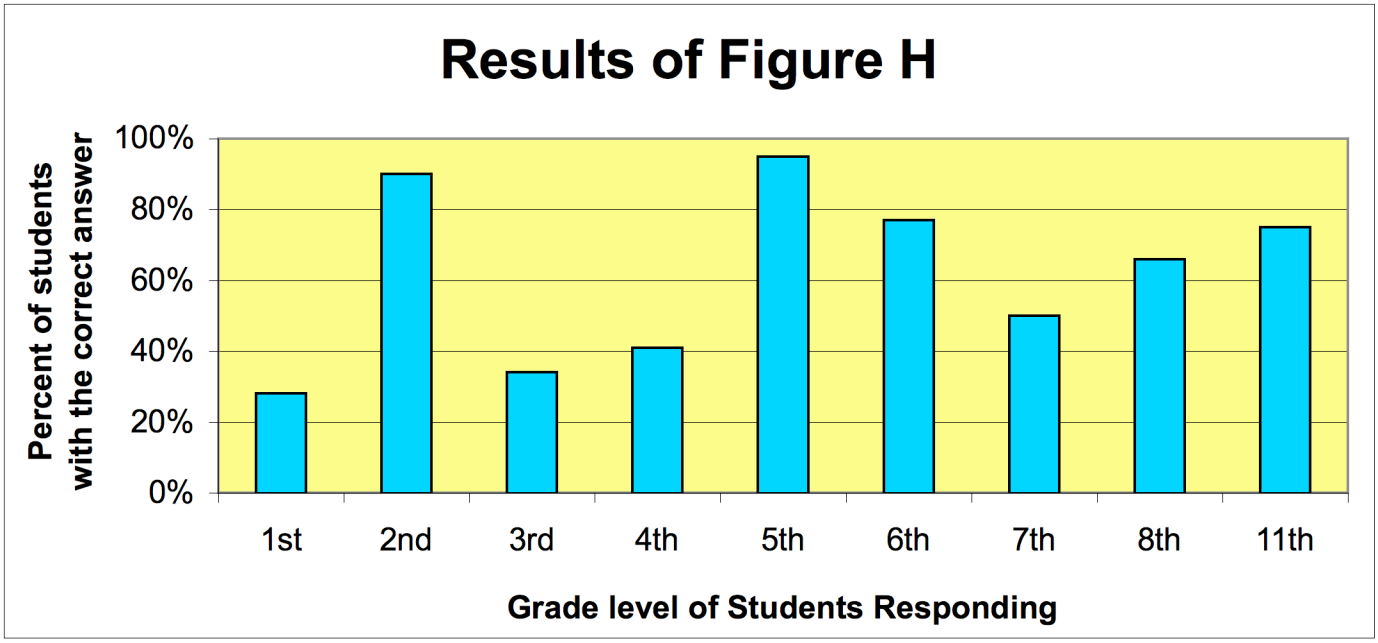
Results for Figure H

Percent Correct by Grade





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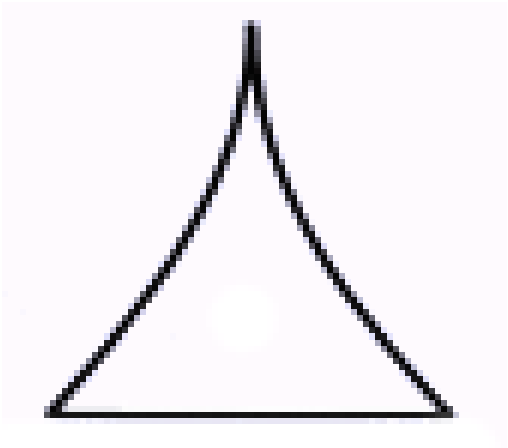
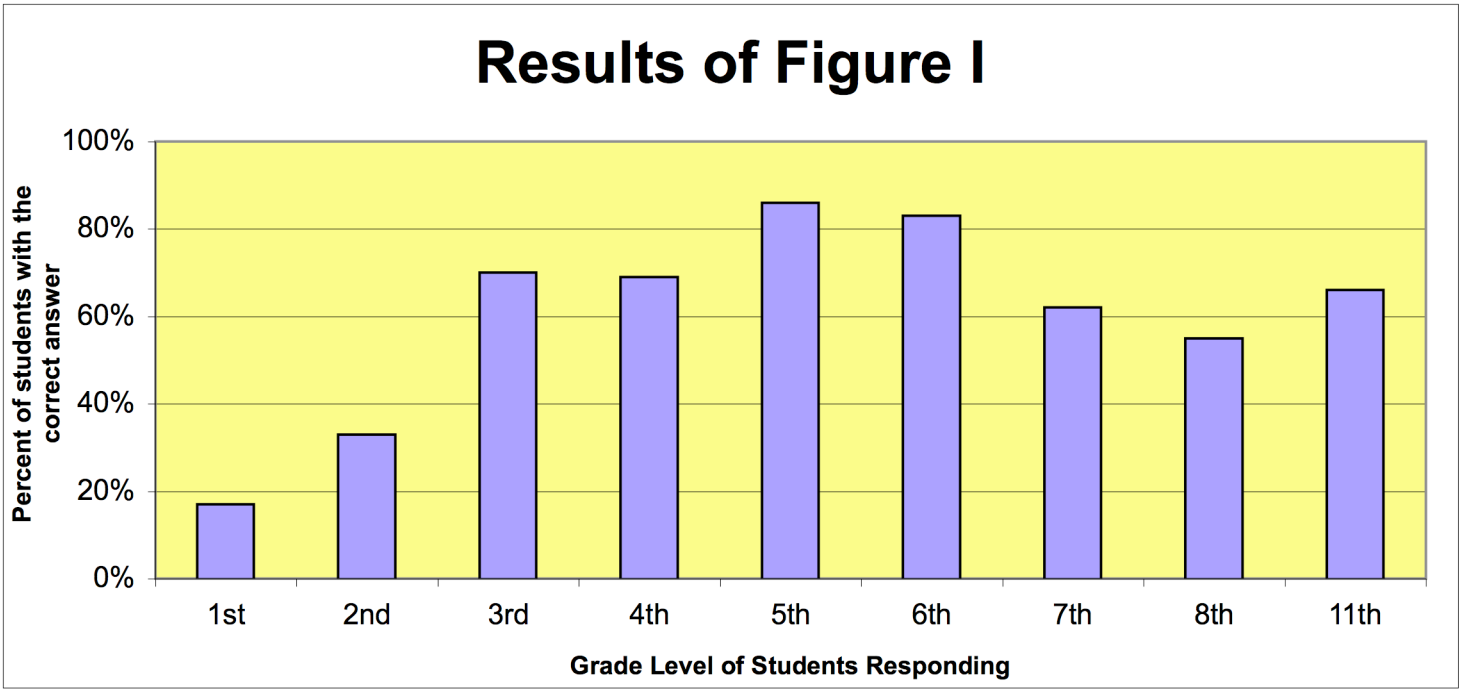
Results for Figure I

Percent Correct by Grade





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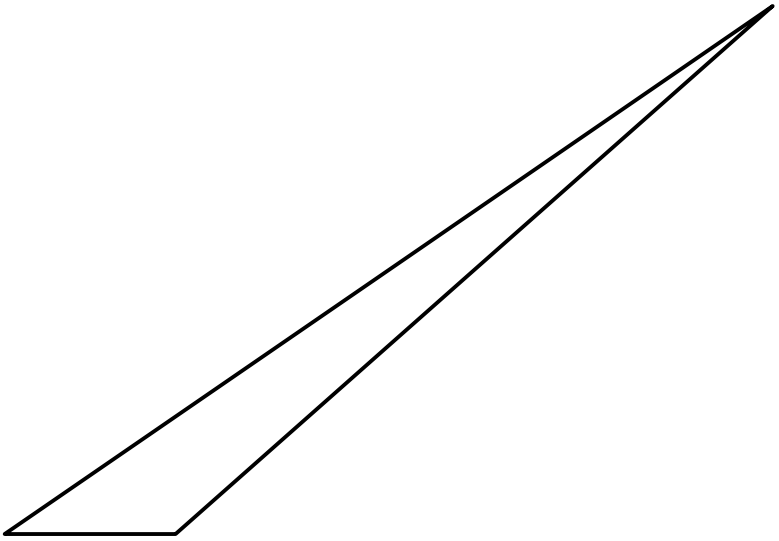
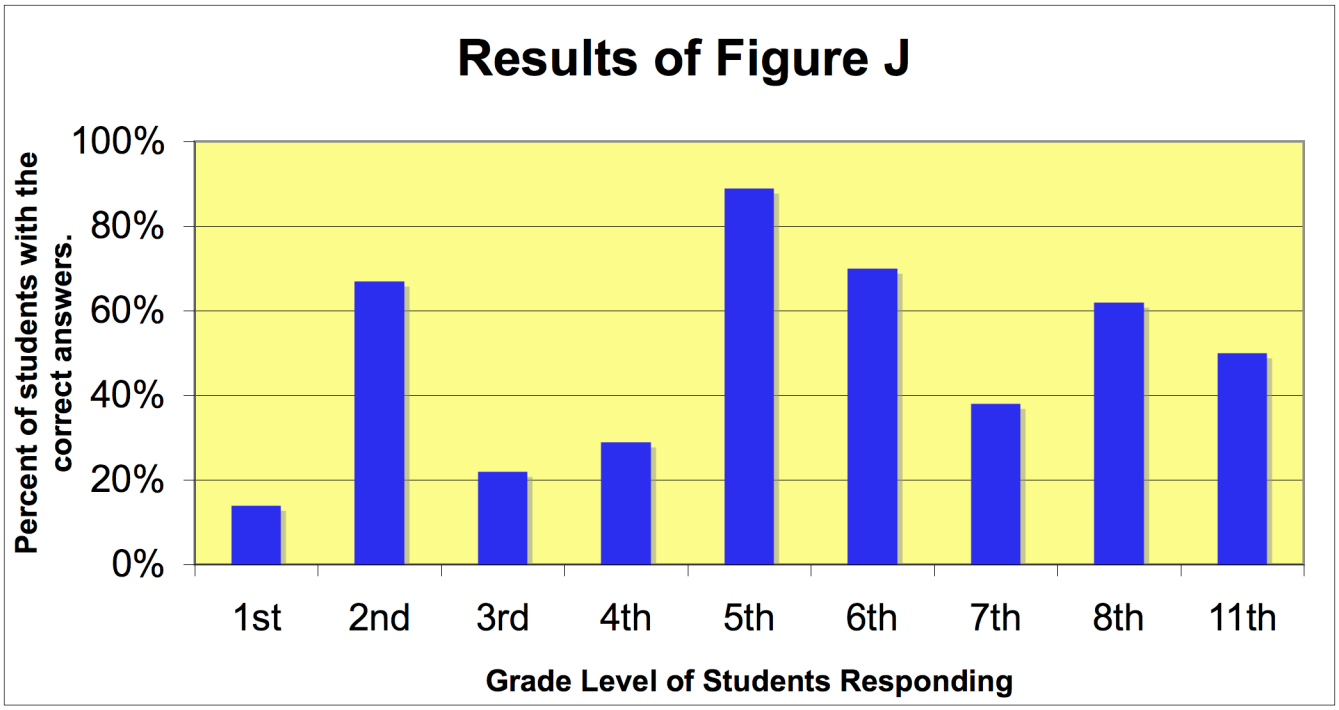
Results for Figure J

Percent Correct by Grade





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Revisiting Your Triangle Definition

Review and revise your definition of a triangle.

Highlight the main ideas you want to emphasize with your students.

Share definitions of triangles.

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MTL Assignment

Administer the Tricky Triangle Task to at least one class in your school.

Tally the results and bring back 4 to 6 student work samples of interest.

We will take time to discuss your findings in January.

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Navigating through Geometry
Suggested Activities

Prekindergarten–Grade 2

- Shapes from Shapes (p. 14)
- Alike and Different (p. 17)
- Name That Block (p. 19)
- Cutting Corners (p. 22)

Grades 3–5

- Build What I've Created (p. 11)
- Thinking About Triangles (p. 15)
- Building Solids (p. 26)

Grades 6–8

- Geodee's Sorting Scheme (p. 13)
- Exploring Triangles (p. 16)

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Big Ideas of Geometry

- Two- and three-dimensional objects can be described, classified and analyzed by their attributes.

- Objects can be oriented in an infinite number of ways. The orienta1tion of an object does not change the other attributes of the object.

- Some attributes of objects (e.g. area, volume, perimeter, surface area) are measurable and can be quantified using unit amounts.

- Objects can be constructed from or decomposed into other objects. In particular, any polygon can be decomposed into triangles.

Development Through the van Hiele Levels

- Level is not affected by biological age.
- Level is affected by degree of experience.
- In order to progress through the levels, instruction must be sequential and intentional.
- When instruction (or materials or vocabulary, etc.) is at an inappropriate level, students will not be able to understand the instruction. They may be able to memorize it, but with no understanding of material.

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