Mathematics Curriculum Guide
Grade 4
Mathematics – Grade 4

Introduction: Mathematical Content Standards

In Grade 4, instructional time should focus on three critical areas: (1) developing understanding and fluency with multi-digit multiplication, and developing understanding of dividing to find quotients involving multi-digit dividends; (2) developing an understanding of fraction equivalence, addition and subtraction of fractions with like denominators, and multiplication of fractions by whole numbers; (3) understanding that geometric figures can be analyzed and classified based on their properties, such as having parallel sides, perpendicular sides, particular angle measures, and symmetry.

1. Students generalize their understanding of place value to 1,000,000, understanding the relative sizes of numbers in each place. They apply their understanding of models for multiplication (equal-sized groups, arrays, area models), place value, and properties of operations, in particular the distributive property, as they develop, discuss, and use efficient, accurate, and generalizable methods to compute products of multi-digit whole numbers. Depending on the numbers and the context, they select and accurately apply appropriate methods to estimate or mentally calculate products. They develop fluency with efficient procedures for multiplying whole numbers; understand and explain why the procedures work based on place value and properties of operations; and use them to solve problems. Students apply their understanding of models for division, place value, properties of operations, and the relationship of division to multiplication as they develop, discuss, and use efficient, accurate, and generalizable procedures to find quotients involving multi-digit dividends. They select and accurately apply appropriate methods to estimate and mentally calculate quotients, and interpret remainders based upon the context.

2. Students develop understanding of fraction equivalence and operations with fractions. They recognize that two different fractions can be equal (e.g., 15/9 = 5/3), and they develop methods for generating and recognizing equivalent fractions. Students extend previous understandings about how fractions are built from unit fractions, composing fractions from unit fractions, decomposing fractions into unit fractions, and using the meaning of fractions and the meaning of multiplication to multiply a fraction by a whole number.

3. Students describe, analyze, compare, and classify two-dimensional shapes. Through building, drawing, and analyzing two-dimensional shapes, students deepen their understanding of properties of two-dimensional objects and the use of them to solve problems involving symmetry.

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.
Mathematics – Grade 4

The assessments and strategies for learning listed on this page are not meant to be an “end all”. Additionally, these are repeated for every grade level curriculum guide with the intent that our students experience some of the same strategies year to year, teacher to teacher. On the following pages, you will find specific usage for some of these.

Assessment for learning provides students with insight to improve achievement and helps teachers diagnose and respond to student needs. (Stiggins, Rick, Judith Arter, Jan Chappuis, Steve Chappuis. Classroom Assessment for Student Learning. Educational Testing Service, 2006.)

Assessment tools include:
- Constructed response items
- Descriptive Feedback (oral/written)
- Effective use of questioning
- Exit slips
- Milwaukee Math Partnership (MMP) CABS
- Observational checklists and anecdotal notes
- Portfolio items
- Projects (PBL)
- Student journals
- Student self-assessment
- Students analyze strong and weak work samples
- Use of Learning Intentions
- Use of rubrics with students
- Use of talk formats and talk moves, p. 55 of the CMSP
- Use of Success Criteria

Benchmark/Universal Screener:
Measures of Academic Progress (MAP)

Summative Assessments:
Wisconsin Knowledge and Concepts Examination (WKCE) or Wisconsin Alternative Assessment (WAA)

Comprehensive literacy is the ability to use reading, writing, speaking, listening, viewing and technological skills and strategies to access and communicate information effectively inside and outside of the classroom and across content areas. (CLP, p.11)

Literacy strategies can help students learn mathematics:
- Concept mapping
- Graphic organizers
- Journaling
- K-W-L
- Literature
- RAFT
- Reciprocal teaching
- Talk moves
- Think Aloud strategy
- Think, Pair, Share strategy
- Three-minute pause
- Two column note taking
- Vocabulary strategies (e.g. Marzano’s 6 steps; Frayer model)
## Operations & Algebraic Thinking

<table>
<thead>
<tr>
<th>Use the four operations with whole numbers to solve problems.</th>
<th>Gain familiarity with factors and multiples.</th>
<th>Generate and analyze patterns.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Interpret a multiplication equation as a comparison, e.g., interpret $35 = 5 \times 7$ as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.</td>
<td>4. Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.</td>
<td>5. Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. For example, given the rule “Add 3” and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.</td>
</tr>
<tr>
<td>2. Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.</td>
<td>3. Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.</td>
<td></td>
</tr>
</tbody>
</table>

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1 See Glossary, Table 2.
### Essential/Enduring Understandings

- Develop an understanding and fluency with multi-digit multiplication.
- Develop an understanding of dividing to find quotients involving multi-digit dividends.
- Use and describe the relationship between factors and multiples. (Multiples can be thought of as the result of skip counting by each of the factors. When skip counting, students should be able to identify the number of factors counted e.g., 5, 10, 15, 20 (there are 4 fives in 20).)
- Create patterns with numbers or shapes that satisfy a given rule and analyze these patterns for their characteristics.

### Assessments

<table>
<thead>
<tr>
<th>Formative Assessment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constructed Response Item (CR); Babysitting Prompt</td>
</tr>
<tr>
<td>Classroom Assessments Based on Standards (CABS)</td>
</tr>
<tr>
<td>4.OA.1-3:</td>
</tr>
<tr>
<td>Grade 4 Algebra 4, 5, 6, 7.</td>
</tr>
<tr>
<td>Grade 4 Number Operations 2, 3, 4, 5</td>
</tr>
<tr>
<td>Grade 5 Number Operations 1, 2</td>
</tr>
<tr>
<td>4.OA.4:</td>
</tr>
<tr>
<td>Grade 3 Number Operations 11</td>
</tr>
<tr>
<td>Grade 5 Number Operations 5, 6</td>
</tr>
<tr>
<td>Grade 6 Number Operations 9, 14</td>
</tr>
<tr>
<td>4.OA.5:</td>
</tr>
<tr>
<td>Grade 4 Algebra 1, 3</td>
</tr>
<tr>
<td>Grade 5 Algebra 1, 13</td>
</tr>
</tbody>
</table>

### Common Misconceptions/Challenges

**4.OA Cluster:** Use the four operations with whole numbers to solve problems.  
**4.OA Standards 1-3**

- Students have a difficult time with problems in context and identifying the appropriate operation to use. Time needs to be spent on the meaning of addition, subtraction, multiplication and division. For example, stories of “groups of groups” are typically multiplication problems. However, students can use repeated addition to solve multiplication contexts and based on readiness should not be penalized but supported to use more efficient methods.
- Students view the equal sign as “the answer is” rather than “is the same as.” They need a foundation in understanding this idea of a quantitative and balanced relationship—that the expressions on either side of the equal sign represent the same quantity. When students lack understanding of the equality relationship they find it difficult to work with equations in which there are unknowns, either on one or both sides of the equal sign.
- Many students know their “basic facts” but when faced with more challenging problems, use procedures without meaning. For example
Mathematics – Grade 4

for the problem 56 x 23, a student might treat each numeral as a digit not considering that the 2 in the 23 really means 20 and the 5 in 56 really means 50. When student multiply without meaning and regroup they often misalign their products and do not stop to consider the reasonableness of their final products.

- Students look at digits while dividing, rather than the actual numeral that is being divided. A common error for 3208 divided by 8 may sound like, “Eight doesn’t go into 3, but it goes into 32 four times. Eight doesn’t go into 0, but it goes into 8 one time. So, eight goes into 3,208 forty-one times.” The answer of 41 is unreasonable. Students should know that 8 x 41 will not get a product of 3,208, and need to test the reasonableness of answers, use estimation strategies, and move beyond rules without meaning.
- Students struggle with our traditional algorithm for division. The procedure contains multiple steps and students make many errors. The algorithm treats the dividend as a set of digits rather than an entire numeral and students are taught to ignore place value. This impedes their ability to think about whether an answer even makes sense.
- Students inappropriately represent a remainder in the context of word problems, or ignore them all together.

4.OA Cluster: Gain familiarity with factors and multiples.
4.OA Standard 4
- When listing multiples of numbers, students often wonder about listing the number itself. Emphasize that the smallest multiple is the number itself. Connect this to the factor pair of the number itself and the number one, e.g. (a, 0), and discuss the identity property.
- Some students may think that larger numbers have more factors. For instance, students may say 97 has more factors than 90. The factors for 97 are (97, 1), the number is in fact prime. Whereas the factors for 90 are (90,1) (45, 2) (30,3) (18,5) (6,15) and (9, 10), the number is in fact composite.

4.OA Cluster: Generate and analyze patterns.
4.OA Standard 5
- Students are often unaware of the different types of patterns: growing patterns (where a pattern changes by increments i.e. +2 +3 +4) and repeating patterns (where a pattern changes in the same way throughout i.e. +2).
- Students have not had experience with identification of the “core unit” in a given pattern, and may have been limited to pattern extension of simply, “What comes next?”

Instructional Practices

4.OA Cluster: Use the four operations with whole numbers to solve problems.
4.OA Standards 1-3
- Record a student’s method for solving a problem using numbers both horizontally and vertically where applicable, e.g. during the summary of a lesson when students are sharing their methods.
- Use the partial products strategy in a launch and exploration. Have students model the operation of the multiplier on both tens and ones values of the multiplicand, considering the value of the whole numbers rather than the digits.[e.g., 42 x 36 = (40 x 30) + (40 x 6) + (2 x 30) + (2 x 6)]. Have students demonstrate and share their solutions during the summary.
- Encourage and model estimation strategies, in launch and summary, for all four operations, before students solve. [e.g., 65 apples shared equally among 5 friends, is a division situation for 65 divided by 5. A student should be able to say, “I know 10 x 5 is 50, so my answer
Mathematics – Grade 4

must be more than 10.”

- **Launch** word problems involving both the partitive model for division and the measurement model for division.
- **Launch** division problems in context such that the remainder impacts a final answer. For instance, if 54 students are going on a field trip, and 20 students fit on each bus, then how many buses need to be ordered? The solution of 2 remainder 14 does not work in this case, 3 buses would need to be ordered for the trip.

4.OA Cluster: Gain familiarity with factors and multiples.
4.OA Standard 4
- Use a 100’s chart to identify multiples of a given number as patterns. This can be used during an exploration and the mathematics should be clarified during the summary of a lesson.
- Students **explore** whether numbers are prime or composite by building rectangles (arrays) with the given area and finding which numbers have more than two rectangles (e.g. 7 can be made into only 2 rectangles, 1 x 7 and 7 x 1, therefore it is a prime number)
- Finding factors of the number
- Multiples can be thought of as the result of skip counting by each of the factors. When exploring skip counting, students should be able to identify the number of factors counted e.g., 5, 10, 15, 20 (there are 4 fives in 20).

4.OA Cluster: Generate and analyze patterns.
4.OA Standard 5
- Utilize the counting sequence as a growth pattern during the launch of a lesson, allow students to explore with the counting sequence and summarize connections.
- Patterns and rules are related. A pattern is a sequence that repeats the same process over and over. A rule dictates what that process will look like. Students **explore** different patterns to find rules, identify features in the patterns, and justify the reason for those features in a summary.

Differentiation
4.OA Cluster: Use the four operations with whole numbers to solve problems.
4.OA Standards 1-3
- Model the mathematical situation using manipulatives for smaller quantities. $8 + 8 = 10 + n$, before moving to larger quantities in the expressions.
- Replace the “box” in an equation with a letter as soon as students recognize the difference between letters and numbers in mathematics.
- Estimation skills include identifying when estimation is appropriate, determining the level of accuracy needed, selecting the appropriate method of estimation, and verifying solutions or determining the reasonableness of situations using various estimation strategies.
  - Estimation strategies include, but are not limited to:
    - Use front-end estimation with adjusting (using the highest place value and estimating from the front end, making adjustments to the estimate by taking into account the remaining amounts),
    - Use clustering around an average (when the values are close together an average value is selected and multiplied by the number
Mathematics – Grade 4

- Use rounding and adjusting strategies (students round down or round up and then adjust their estimate depending on how much the rounding affected the original values).
- Use friendly or compatible numbers such as factors (students seek to fit numbers together - e.g., rounding to factors and grouping numbers together that have round sums like 100 or 1000).
- Use benchmark numbers that are easy to compute (student’s select close whole numbers for fractions or decimals to determine an estimate).

4.OA Cluster: Gain familiarity with factors and multiples.
4.OA Standard 4
- Begin with numbers that only have one pair of factors, and then move to those that have several pairs.

4.OA Cluster: Generate and analyze patterns.
4.OA Standard 5
- Use patterns to; identify the next in a sequence, complete missing items in a sequence, and move to items that would occur later in a sequence.
- Students need practice with multiple representations of patterns and functions using words, tables and graphs.
- Vary given rules for patterns for students; e.g. “Add 3” and “Multiply 4”.
- Use meaningful contexts for patterns and functional relationships; money spent on candy, ingredients for a recipe, blocks used to build towers, time needed to run a race, fuel needed for a vacation.

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**Literacy Connections**

<table>
<thead>
<tr>
<th>Literacy Strategies:</th>
<th>Academic Vocabulary</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model the language of the relationship in patterns and encourage students to describe the relationship. E.g., The amount of money I spend depends on how much candy I buy.</td>
<td>equation</td>
</tr>
<tr>
<td>Model the correct vocabulary specific to each manipulative and plan lessons in which students use these manipulatives.</td>
<td>estimation</td>
</tr>
<tr>
<td>Use a think-a-loud with problems in context.</td>
<td>even numbers</td>
</tr>
<tr>
<td>Provide a variety of rich and realistic word problems.</td>
<td>factor and factor pairs</td>
</tr>
<tr>
<td>Have students write and solve their own word problems.</td>
<td>multiple</td>
</tr>
</tbody>
</table>

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**Academic Vocabulary**

- equation
- estimation
- even numbers
- factor and factor pairs
- multiple
- odd numbers
- prime and composite numbers
- product
- rule
### Mathematics – Grade 4

**Literature Suggestions:**

<table>
<thead>
<tr>
<th>Resources</th>
</tr>
</thead>
<tbody>
<tr>
<td>- calculators</td>
</tr>
<tr>
<td>- picture models and drawings</td>
</tr>
<tr>
<td>- number line</td>
</tr>
<tr>
<td>- area models; graph paper, square tiles, base ten blocks</td>
</tr>
<tr>
<td>- patterning manipulatives; counters, blocks</td>
</tr>
<tr>
<td>- <a href="http://illuminations.nctm.org/LessonDetail.aspx?ID=L620">http://illuminations.nctm.org/LessonDetail.aspx?ID=L620</a>; The Factor Game engages students in a friendly contest in which winning strategies involve distinguishing between numbers with many factors and numbers with few factors.</td>
</tr>
<tr>
<td>- <a href="https://www.pbs.org/teachers/mathline/lessonplans/atmp/snake/snake_procedure.shtm">https://www.pbs.org/teachers/mathline/lessonplans/atmp/snake/snake_procedure.shtm</a>; Students will use given rules to generate several stages of a snake pattern and will be able to predict the outcome for any stage.</td>
</tr>
</tbody>
</table>
Mathematics – Grade 4

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Number & Operations in Base Ten¹

<table>
<thead>
<tr>
<th>Generalize place value understanding for multi-digit whole numbers.</th>
<th>Use place value understanding and properties of operations to perform multi-digit arithmetic.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. For example, recognize that (700 ÷ 70 = 10) by applying concepts of place value and division.</td>
<td>4. Fluently add and subtract multi-digit whole numbers using the standard algorithm.</td>
</tr>
<tr>
<td>2. Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using &gt;, =, and &lt; symbols to record the results of comparisons.</td>
<td>5. Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.</td>
</tr>
<tr>
<td>3. Use place value understanding to round multi-digit whole numbers to any place.</td>
<td>6. Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.</td>
</tr>
</tbody>
</table>

¹Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000
### Essential/Enduring Understanding

- Students generalize their understanding of place value to 1,000,000, understanding the relative sizes and value of numbers in each place.

- Develop fluency with efficient procedures for multiplying and dividing whole numbers; understand and explain why the procedures work based on place value and properties of operations; and use them to solve.

- Students efficiently and effectively add and subtract multi-digit whole numbers.

### Assessments

#### Formative Assessment

- Teacher observation with anecdotal notes
- Constructed Response Item (CR)
- Student work
- Student discussions
- Math Journal
- Rubric
- Checklist
- Progress monitoring
- Classroom Assessments Based on Standards (CABS)

<table>
<thead>
<tr>
<th>4.NBT.1-3:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grade 3 Number Operations 1, 2, 6</td>
</tr>
<tr>
<td>Grade 5 Number Operations 4</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>4.NBT.4-6:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grade 4 Number Operations 3, 4</td>
</tr>
<tr>
<td>Grade 5 Number Operations 1, 2, 3</td>
</tr>
</tbody>
</table>

### Common Misconceptions/Challenges

**4.NBT Cluster: Generalize place value understanding for multi-digit whole numbers.**

**4.NBT Standards 1-3**

- Zero is perceived as a number without value and is often thought of as a place holder. This misconception is carried into work with larger numbers and is often misleading for students and impedes their effective use of place value structure in decomposition and operations. Use expanded form to explain $1,207 = 1,000(1 \text{ thousand}) + 200(2 \text{ hundred}) + 0(0 \text{ tens}) + 7(7 \text{ units})$

- Students misuse language when speaking about numbers. For example, $1,207$ is incorrectly referred to as “one thousand, two hundred and seven”, which actually represents $1,200.07$

**4.NBT Cluster: Use place value understanding and properties of operations to perform multi-digit arithmetic.**

**4.NBT Standards 4-6**

- Students confuse the idea of commutativity, or the commutative property in addition and multiplication and incorrectly think that it can be used in subtraction and divisions contexts as well.

- Students think about each digit in addition and subtraction, rather than looking at the total number. $48 + 12$ is seen incorrectly
Mathematics – Grade 4

as 8 + 2 and 4 + 1, which is really 40 + 10 and 8 + 2 , or in subtraction as 67-43 as 7 -3 and then 6-4, which is really 60 – 40 and 7 - 3.
- Regrouping when adding.
- Renaming when subtracting.
- Students are taught key words to solve problems that do not fit beyond join and separate. Therefore, making computational errors by choosing incorrect operations for solving.
- Students are often not exposed to the meaning of multiplication and division, and therefore get lost in procedure. When taught meaning and with experience in models for multiplication and division students are often more secure in their understanding.

Instructional Practices

4.NBT Cluster: Generalize place value understanding for multi-digit whole numbers.

4.NBT Standards 1-3
- Explore connections between counting by ones to counting by groups and singles (tens and ones, or hundreds, tens and ones), and summarize in order to develop an understanding of place value.
- Launch a variety of problem types and structures for students to think beyond addition as only joining and subtractions as only taking away.
- Utilize an open number line for addition and subtraction during an exploration.
- Work with a 100s chart to increase understanding of place value. E.g., find 53, find the number that is 10 more or 10 less. Explore how many rows of 10 are in 100, make explicit connections to multiplicative thinking hence moving beyond additive models.
- Decompose numbers and use this work for all four operations.

4.NBT Cluster: Use place value understanding and properties of operations to perform multi-digit arithmetic.

4.NBT Standards 4-6
- Launch, explore and summarize multiple strategies to solve problems. E.g., 25 x 24

<table>
<thead>
<tr>
<th>20</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>400</td>
</tr>
<tr>
<td>4</td>
<td>80</td>
</tr>
<tr>
<td>480</td>
<td>+ 120</td>
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</tbody>
</table>
Launch the area model for multiplication. Have students use base-ten blocks and grid paper to construct arrays to represent problems. E.g., 16 x 14 = 224.

Using the area model, students first verbalize their understanding:

- 10 x 10 is 100
- 4 x 10 is 40
- 10 x 6 is 60, and
- 4 x 6 is 24.

They use different strategies to record this type of thinking.

Model and utilize multiple algorithms for multiplication and division.

After developing an understanding of using arrays to divide, students explore and begin to use a more abstract model for division. E.g., 150 divided by 6.

150 divided by 6 equals 25

Differentiation

| 4.NBT Cluster: Generalize place value understanding for multi-digit whole numbers.  |
| 4.NBT Standards 1-3                                                   |
| - Discuss the meaning of digits beyond what place they are in.           |
| - Utilize expanded form as a tool for students to explain place value and rounding. |
Mathematics – Grade 4

4.NBT Cluster: Use place value understanding and properties of operations to perform multi-digit arithmetic.

4.NBT Standards 4-6

- Sharing of strategies is a powerful way to deepen student understanding of concepts. They are able to see strategies used by peers and often more readily understand an alternate algorithm than one they are struggling with.
- Utilize manipulatives such as base ten blocks as students move from concrete modeling to more abstract thinking.

**Literacy Connections**

<table>
<thead>
<tr>
<th>Literacy Strategies:</th>
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</tr>
</thead>
<tbody>
<tr>
<td>Utilize a K-W-L for the big ideas in both clusters under this domain to access prior knowledge from grade 3 and begin an extended unit of study on numbers.</td>
<td>algorithm</td>
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<tr>
<td></td>
<td>area model</td>
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<td>difference</td>
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<td></td>
<td>digit</td>
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<td>expanded form</td>
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<td>fewer</td>
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<td>less</td>
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<td>more</td>
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<td></td>
<td>partial products</td>
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<tr>
<td></td>
<td>round(ing)</td>
</tr>
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<td></td>
<td>standard form</td>
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<td></td>
<td>sum</td>
</tr>
</tbody>
</table>

**Literature Suggestions:**


**Resources**

- Place value boxes
- Place value mats
- Place value flip charts
- Number cards
- Base ten blocks
- Tens frames
- Hundreds flats
- Calculators
### Mathematics – Grade 4

#### Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

<table>
<thead>
<tr>
<th><strong>Number &amp; Operations—Fractions</strong>¹</th>
<th><strong>4.NF</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Extend understanding of fraction equivalence and ordering.</strong></td>
<td><strong>Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.</strong></td>
</tr>
</tbody>
</table>
| 1. Explain why a fraction \( \frac{a}{b} \) is equivalent to a fraction \( \frac{(n \times a)}{(n \times b)} \) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions. | 3. Understand a fraction \( \frac{a}{b} \) with \( a > 1 \) as a sum of fractions \( \frac{1}{b} \).  
   a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.  
   b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model.  
   c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction. | 5. Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100.²  
   For example, express \( \frac{3}{10} \) as \( \frac{30}{100} \), and add \( \frac{3}{10} + \frac{4}{100} = \frac{34}{100} \). |
| 2. Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as \( \frac{1}{2} \). Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols \( >, =, \) or \( < \), and justify the conclusions, e.g., by using a visual fraction model. | | 6. Use decimal notation for fractions with denominators 10 or 100. For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram. |
| | | 7. Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols \( >, =, \) or \( < \), and justify the conclusions, e.g., by using a visual model. |

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² For example, express \( \frac{23}{100} \) as \( \frac{230}{1000} \), and \( \frac{3}{10} + \frac{4}{100} = \frac{34}{100} \).
4. Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.
   a. Understand a fraction $a/b$ as a multiple of $1/b$. For example, use a visual fraction model to represent $5/4$ as the product $5 \times (1/4)$, recording the conclusion by the equation $5/4 = 5 \times (1/4)$.
   b. Understand a multiple of $a/b$ as a multiple of $1/b$, and use this understanding to multiply a fraction by a whole number. For example, use a visual fraction model to express $3 \times (2/5)$ as $6 \times (1/5)$, recognizing this product as $6/5$. (In general, $n \times (a/b) = (n \times a)/b$.)
   c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. For example, if each person at a party will eat $3/8$ of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?
Mathematics – Grade 4

1 Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, 100.

2 Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general. But addition and subtraction with unlike denominators in general is not a requirement at this grade.
### Mathematics – Grade 4

<table>
<thead>
<tr>
<th>Essential/Enduring Understandings</th>
<th>Assessments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Develop an understanding of fraction equivalence.</td>
<td><strong>Formative</strong></td>
</tr>
<tr>
<td>Develop an understanding of addition and subtraction of fractions with like denominators, and multiplication of fractions by whole numbers.</td>
<td><strong>Constructed Response Item (CR): Green and Black Blocks Prompt</strong></td>
</tr>
<tr>
<td></td>
<td><strong>Student discussions:</strong> Have students explain why one fraction is larger than another; use benchmark fractions, unit fractions, fractions with the same numerator and different denominators, fractions with the same denominator and different numerators, and fractions missing one unit.</td>
</tr>
<tr>
<td></td>
<td><strong>Classroom Assessments Based on Standards (CABS)</strong></td>
</tr>
<tr>
<td></td>
<td>4.NF.1-2:</td>
</tr>
<tr>
<td></td>
<td>Grade 4 Number Operations 8, 9, 14</td>
</tr>
<tr>
<td></td>
<td>Grade 5 Number Operations 10</td>
</tr>
<tr>
<td></td>
<td>4.NF.3-4:</td>
</tr>
<tr>
<td></td>
<td>Grade 4 Measurement 9</td>
</tr>
<tr>
<td></td>
<td>Grade 6 Number Operations 6, 11, 12</td>
</tr>
<tr>
<td></td>
<td>4.NF.5-7:</td>
</tr>
<tr>
<td></td>
<td>Grade 4 Number Operations 12</td>
</tr>
<tr>
<td></td>
<td>Grade 5 Number Operations 9, 10</td>
</tr>
</tbody>
</table>

### Common Misconceptions/Challenges

<table>
<thead>
<tr>
<th>4.NF Cluster: Extend understanding of fraction equivalence and ordering.</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.NF Standards 1-2</td>
</tr>
<tr>
<td>The concept of equivalence has long been taught as “reducing” leading students to assume that the fraction is getting smaller. Therefore, they think the fractions are changing, or decreasing in size, even though they are equivalent. Students believe that fractions that are reduced are different then the fraction they began with, thus disregarding the idea of equivalence. E.g., “If I reduce 12/18 to 6/9, and then reduce 6/9 to 2/3, the new fraction is 2/3.” The fraction is not “new”, but equivalent.</td>
</tr>
<tr>
<td>Students incorrectly order fractions choosing 1/4 as larger than 1/3. They utilize whole number concepts when working with fractions and do not fully understand that the larger the denominator, the smaller each of the equal parts in the whole.</td>
</tr>
<tr>
<td>Students have a difficult time justifying size of fractions. For example, they might think that 7/8 and 5/6 are equal because they are both missing one “piece.” They need work in understanding that each of those fractions is missing a unit fraction and since sixths are larger than eights, 7/8 is larger than 5/6 because it is missing a smaller unit.</td>
</tr>
</tbody>
</table>

| 4.NF Cluster: Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers. |
Mathematics – Grade 4

4.NF Standards 3-4

- In comparing unit fractions 1/4 and 1/6, for instance, students often learn that “the larger the denominator the smaller the fraction.” They carry this with them when comparing fractions that are greater than units, incorrectly assuming that 2/4 is larger than 5/6 because of the size of the denominator.
- Students find it difficult to divide some objects equally (circles, muffins, for example) which may result in misunderstandings about the size of a fraction, especially if they have not partitioned the object/area into equal sized portions. Students generalize whole number concepts when computing with fractions. For example, they add numerators together, seeing them as whole number, and do the same for the denominators. They are seeing each part of the fraction as a single-digit whole number.
- A rush to procedure leads to a lack of conceptual understanding between mixed numbers and fractions. For example: 12/5 is 2 and 2/5 because it is 5/5 + 5/5 + 2/5. Also, 4 3/8 is 8/8 + 8/8 + 8/8 + 8/8 + 3/8. Students often do not understand this when they practice multiplication and addition strategies rooted in whole number understanding to move between the fraction representations.

4.NF Cluster: Understand decimal notation for fractions, and compare decimal fractions.

4.NF Standards 5-7

- Often students incorrectly represent equality of fractions and decimals, perhaps thinking 1/2 is .2 or 5/8 is .5, they are not considering equivalence and place value.
- Students have a difficult time providing the word form for fractions and decimals, e.g. six-tenths, thirty-seven-hundredths.
- There is a misconception that the longer the number, the greater the value. Students simply look at the number of digits and disregard the decimal point.
- In computing with decimals, students apply whole number concepts to decimals. They misalign digits and lack understanding of where decimal points should be placed, thus demonstrating a lack of understanding around place value.

Instructional Practices

4.NF Cluster: Extend understanding of fraction equivalence and ordering.

4.NF Standards 1-2

- During a launch, explore and summarize, utilize models that lend themselves to equal pieces, or fractions being shared. A circle is a difficult model to use when representing any fractional part other than halves, fourths and eighths.
- Students need experiences dividing many shapes and representations for fractions, in order to determine which shapes to use for particular contexts. Launch, explore and summarize, examples for parts of a whole and examples for parts of a set.
- Utilize estimation strategies when working with fractions in operations, as part of a launch, explore and summary.

4.NF Cluster: Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

4.NF Standards 3-4

- Explore the idea of fair sharing and the interconnectedness to fractions as dividing two numbers.
- Through exploration, students begin to notice connections between the models and fractions in the way both the parts and wholes are counted and begin to generate a rule for writing equivalent fractions.
  
  \[ \frac{1}{2} \times \frac{2}{2} = \frac{2}{4}. \]
A unit fraction is a fraction with a numerator of one. When students explore fractions other than unit fractions, such as 2/3, they should be able to decompose the non-unit fraction into a combination of several unit fractions. Example: 2/3 = 1/3 + 1/3 also 2/3 = 2(1/3)

Use the number line to represent and order fractions during an exploration, using the benchmark of 1/2.

4.NF Cluster: Understand decimal notation for fractions, and compare decimal fractions.
4.NF Standards 5-7

- Explore for understanding, that a decimal amount is a sum of a whole number and a number less than one. E.g., forty-three and twenty-nine hundredths, is forty-three wholes and twenty-nine hundredths is less than one. This may look like 40.19 + 3.10 or 43 + .29. Students should communicate this during a summary.
- Explore using a 10 x 10 block, not as 100, but as a whole, with each part of the grid representing one-hundredth.
**Mathematics – Grade 4**

**Differentiation**

4.NF Cluster: Extend understanding of fraction equivalence and ordering.
4.NF Standards 1-2
- Utilize multiple models and representations for fractions, linear, measurement, parts of whole, parts of set, fraction as division.
- Utilize fraction strips as a fraction manipulative with students.

4.NF Cluster: Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.
4.NF Standards 3-4
- Practice counting by fractions with students.

4.NF Cluster: Understand decimal notation for fractions, and compare decimal fractions.
4.NF Standards 5-7
- Decimals used in computation should connect to real life contexts beyond money, including speed, measures (distance, liquid, linear) and the radio dial.

**Literacy Connections**

**Literacy Strategies:**
- Use the language of decimals appropriately. For example, 2.47 is “two and forty-seven hundredths,” not “two point forty-seven” or “two decimal forty-seven.”
- Avoid the language of “reduce” or “simplify” as they can mislead students, use “rename” instead.
- Avoid the language of “improper” when referring to a fraction with a larger numerator than denominator, rather expose students to these types of problems and discuss the meaning of the fractions.

**Academic Vocabulary**
- benchmark fraction
- denominator
- equivalence, equivalent
- hundredth
- mixed number
- numerator
- tenth
- unit fraction

**Literature Suggestions:**

**Resources**
Mathematics – Grade 4

- number lines
- fraction strips, fraction bars/tiles, circular fraction models
- rulers with marking of ½, ¼, ⅛
- grid paper
- 10 x 10 squares on grid
- decimal place-value mats
- coins
- base ten blocks
- [http://illuminations.nctm.org/LessonDetail.aspx?id=L861](http://illuminations.nctm.org/LessonDetail.aspx?id=L861) A Meter of Candy- students develop and reinforce their understanding of hundredths as fractions, decimals and percentages. They make and connect a set and linear model to produce area models.
- [http://illuminations.nctm.org/activitydetail.aspx?id=80](http://illuminations.nctm.org/activitydetail.aspx?id=80) Students create equivalent fractions by dividing and shading squares or circles, and match each fraction to its location on the number line.
### Measurement & Data

#### 4.MD

<table>
<thead>
<tr>
<th><strong>Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.</strong></th>
<th><strong>Represent and interpret data.</strong></th>
<th><strong>Geometric measurement: understand concepts of angle and measure angles.</strong></th>
</tr>
</thead>
</table>
| 1. Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. *For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in.* Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), … | 4. Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots. *For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.* | 5. Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement:  
   a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a “one-degree angle,” and can be used to measure angles.  
   b. An angle that turns through \( n \) one-degree angles is said to have an angle measure of \( n \) degrees. |
| 2. Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using |  | 6. Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure. |
## Mathematics – Grade 4

<table>
<thead>
<tr>
<th>Diagrams such as number line diagrams that feature a measurement scale.</th>
</tr>
</thead>
<tbody>
<tr>
<td>3. Apply the area and perimeter formulas for rectangles in real world and mathematical problems. For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.</td>
</tr>
<tr>
<td>7. Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.</td>
</tr>
</tbody>
</table>

### Enduring/Essential Understandings

- Interpret and use appropriate measurements with operators in context.
- Understand that area and perimeter are measurements and how they connect to realistic situations.
- Recognize that angle measures can be analyzed as part of geometric shapes.

### Assessments

#### Formative

- **Constructed Response Item (CR):** Canoeing Prompt, Lemonade Prompt
- **Student work:** Have students participate in an angle hunt and determine whether angles are greater or less than 90 degrees, using the corner of a paper, before any protractor work.
- **Classroom Assessments Based on Standards (CABS)**
  - **4.MD.1-3:**
    - Grade 4 Measurement 1-8, 10, 11
    - Grade 5 Measurement 7
  - **4.MD.4:**
    - Grade 3 Measurement 12
    - Grade 4 Measurement 3
    - Grade 5 Measurement 8, 9
  - **4.MD.5-7:**
    - Grade 5 Measurement 9
    - Grade 6 Geometry 9

### Common Misconceptions/Challenges

Math_Curriculum Guide Gr 4_07.28.11_v1
4.MD Cluster: Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.

4.MD Standards 1-3
- Students incorrectly think that there is a pattern or connection between the numerical measurements of area and perimeter.
- Students may struggle using the operations to convert and move between units of measure.

4.MD Cluster: Represent and interpret data.
4.MD Standard 4
- Students struggle with ordering fractions with different denominators, and therefore may incorrectly order and label a line plot.
- Misrepresentation of data on a line plot often occurs as students initially learn how to interpret and analyze data.

4.MD Cluster: Geometric measurement: understand concepts of angle and measure angles.
4.MD Standards 5-7
- The diagram below will help students understand that an angle measurement is not related to an area since the area between the 2 rays is different for both circles yet the angle measure is the same.

Instructional Practices
4.MD Cluster: Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.
4.MD Standards 1-3
- While students are expected to use formulas to calculate area and perimeter of rectangles, they need to understand and be able to communicate their understanding in a summary of why the formulas work.

4.MD Cluster: Represent and interpret data.
4.MD Standard 4
- Example: Launch the following problem and representation.
- Ten students in Room 31 measured their pencils at the end of the day. They recorded their results on the line plot below.

Possible questions to ask during an explore and/or...
4.MD Cluster: Geometric measurement: understand concepts of angle and measure angles.

4.MD Standards 5-7
- Before students begin measuring angles with protractors, they need to **explore** with benchmark angles.
- Students transfer their understanding that a 360º rotation about a point makes a complete circle to recognize and sketch angles that measure approximately 90º and 180º. They extend this understanding and recognize and sketch angles that measure approximately 45º and 30º.
- They use appropriate terminology (acute, right, and obtuse) to describe angles and rays (perpendicular).

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**summary:**

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<td>3 ½”</td>
<td>4”</td>
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<tr>
<td>4 ¼”</td>
<td>5 1/8”</td>
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<tr>
<td>5 1/2”</td>
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</tbody>
</table>

What is the difference in length from the longest to the shortest pencil?

If you were to line up all the pencils, what would the total length be?

If the 5 1/8” pencils are placed end to end, what would be their total length?

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**Differentiation**

4.MD Cluster: Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.

4.MD Standards 1-3
- **Estimation strategies for number operations can aide in measurement conversion. Determine reasonableness of answers.**
- To develop a richer understanding of area and perimeter, have students explore constant area/changing perimeter contexts, constant perimeter/changing area contexts and move beyond whole number units to fractions and decimals.

4.MD Cluster: Represent and interpret data.

4.MD Standard 4
- Provide students with examples of data from local resources such as school data, district data and data interpretation in magazines, newspapers, and other classroom materials (e.g., science and social studies programming.)
- Provide students with data and have them collect data to represent using a line plot.

4.MD Cluster: Geometric measurement: understand concepts of angle and measure angles.

4.MD Standards 5-7
- Conduct an angle hunt in the classroom and around the school building in order to determine the relevance of the 90 degree angle in building construction.
### Literacy Connections

**Literacy Strategies:**
- Have students create concept maps for both area and perimeter.
- Model and have students use appropriate terminology (acute, right, and obtuse) to describe angles and rays (perpendicular).

### Academic Vocabulary
- angle
- arc
- area
- centimeter (cm)
- degree
- distance
- feet (ft)
- gram (g)
- hour (hr)
- inch (in)
- kilogram (kg)
- kilometer (km)
- line plot
- liter (l)
- mass
- meter (m)
- milliliter (ml)
- minute (min)
- ounce (oz)
- perimeter
- pound (lb)
- protractor
- range
- second (sec)
- volume
- yard (yd)

### Literature Suggestions:

### Resources
- clocks, stop watches
- protractors, compasses
- balance scales
- rulers, yardsticks, tape measures
- *thermometers*
- *grid paper*
- *beakers, graduated cylinders*
- *weights*
Mathematics – Grade 4

Mathematical Practices
1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Geometry

4.G

Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

1. Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.

2. Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.

3. Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.
### Essential/Enduring Understandings
- Understand that geometric figures can be analyzed and classified based on their properties, such as having parallel sides, perpendicular sides, particular angle measures, and symmetry.

### Assessments
**Formative**
- Student work and discussions on the Tricky Triangle task (found on the MMP website)
- Classroom Assessments Based on Standards (CABS)
  - 4.G.1-3:
    - Grade 3 Geometry 5
    - Grade 4 Geometry 3
    - Grade 5 Geometry 4, 6
    - Grade 6 Geometry 10, 11, 13

### Common Misconceptions/Challenges
**4.G Cluster:** Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

**4.G Standards 1-3**
- Examples of points, line segments, lines, angles, parallelism, and perpendicularity can be seen daily. Students do not easily identify lines and rays because they are more abstract.

![Geometric Figures Diagram]

- Students have a difficult time understanding that there are different types of right triangles. An isosceles right triangle has two or more
### Mathematics – Grade 4

- A congruent sides and a scalene right triangle has no congruent sides.
  - Students simply attribute symmetry to a line down the “center” of an object, not considering mirror images.

### Instructional Practices

#### 4.G Cluster: Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

#### 4.G Standards 1-3

- **Launch** the concept of parallel and perpendicular lines. Two lines are parallel if they never intersect and are always equidistant. Two lines are perpendicular if they intersect in right angles (90°).

- Students should **explore** arrangements of two lines in as different ways to determine that the 2 lines might intersect in one point or may never intersect. Further investigations may be initiated using geometry software. These types of **explorations** lead to a student **summary** on angles.

\[
\begin{align*}
A & \quad F & \quad B \\
C & \quad \quad & \quad D \\
\quad & \quad G
\end{align*}
\]

- Students **explore** with figures which are symmetrical and non-symmetrical. Figures include both regular and non-regular polygons. Folding cut-out figures will help students determine whether a figure has one or more lines of symmetry.

### Differentiation

#### 4.G Cluster: Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

#### 4.G Standards 1-3

- Students may use transparencies with lines to arrange two lines in different ways to determine that the 2 lines might intersect in one point or may never intersect. Further investigations may be initiated using geometry software.

- Use patty paper to fold figures. This will help students determine whether a figure has no, one or more than one line of symmetry.

- Provide students with half of a symmetrical image and have them complete the whole, through illustration or with computer software.
### Mathematics – Grade 4

<table>
<thead>
<tr>
<th>Literacy Connections</th>
<th>Academic Vocabulary</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Literacy Strategies:</strong></td>
<td>• angle(s): acute, obtuse, right</td>
</tr>
<tr>
<td>▪ Have students create a semantic feature analysis grid for quadrilaterals with columns addressing angles and parallel and perpendicular sides.</td>
<td>• line(s)</td>
</tr>
<tr>
<td>▪ line segment(s)</td>
<td>• parallel</td>
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<td>▪ parallel</td>
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<td>• point</td>
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<tr>
<td>▪ point</td>
<td>• ray(s)</td>
</tr>
<tr>
<td>▪ symmetry/symmetrical</td>
<td>• two-dimensional</td>
</tr>
<tr>
<td>Literature Suggestions:</td>
<td>Resources</td>
</tr>
<tr>
<td></td>
<td>Geoboards</td>
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<td>Patty Paper</td>
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<td>Pattern blocks</td>
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<td><a href="http://www.geogebra.org">http://www.geogebra.org</a> (a free software)</td>
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<td><a href="http://www.mmp.uwm.edu">http://www.mmp.uwm.edu</a> (search for tricky triangle task)</td>
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</tbody>
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