This Curriculum Guide is to be used in conjunction with the Pacing Guide and Instructional Design Lesson Plan. The Instructional Design will help with lesson planning using Launch-Explore-Summarize.
Mathematics – High School Geometry Course: Introduction

The fundamental purpose of the course in Geometry is to formalize and extend students’ geometric experiences from the middle grades. Students explore more complex geometric situations and deepen their explanations of geometric relationships moving toward formal mathematical arguments. Important differences exist between this Geometry course and the traditional approach taken in Geometry classes. For example, transformations are emphasized early in this course. The Mathematical Practice Standards apply throughout this course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations.

Critical Area 1: In previous grades, students were asked to draw triangles based on given measurements. They also have prior experience with rigid motions: translations, reflections, and rotations and have used these to develop notations about what it means for two objects to be congruent. In this area, students establish triangle congruence criteria, based on analyses of rigid motions and formal constructions. They use triangle congruence as a familiar foundation for the development of formal proof. Students prove theorems – using a variety of formats – and solve problems about triangles, quadrilaterals, and other polygons. They apply reasoning to complete geometric constructions and explain why they work.

Critical Area 2: Students apply their earlier experience with dilations and proportional reasoning to build a formal understanding of similarity. They identify criteria for similarity of triangles, use similarity to solve problems, and apply similarity in right triangles to understand right triangle trigonometry, with particular attention to special right triangles and the Pythagorean Theorem. Students develop the Laws of Sines and Cosines in order to find missing measures of general (not necessarily right) triangles, building students’ work with quadratic equations done in the first course. They are able to distinguish whether three given measures (angles or sides) define 0, 1, 2 or infinitely many triangles.

Critical Area 3: Students’ experience with two-dimensional and three-dimensional objects is extended to include informal explanations of circumference, area and volume formulas. Additionally, students apply their knowledge of two-dimensional shapes to consider the shapes of cross-sections and the result of rotating two-dimensional shapes to consider the shapes of cross-sections and the result of rotating a two-dimensional object about a line.

Critical Area 4: Building on their work with the Pythagorean Theorem in 8th grade to find distances, students use a rectangular coordinate system to verify geometric relationships, including properties of special triangles and quadrilaterals and slopes of parallel and perpendicular lines, which relates back to work done in the first course. Students continue their study of quadratics by connecting the geometric and algebraic definitions of the parabola.

Standards for Mathematical Practice
1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.
Critical Area 5: In this area students prove basic theorems about circles, such as a tangent line is perpendicular to a radius, inscribed angle theorem, and theorems about chords, secants, and tangents dealing with segment lengths and angle measures. They study relationships among segments on chords, secants, and tangents as an application of similarity. In the Cartesian coordinate system, students use the distance formula to write the equation of a circle when given the radius and the coordinates of its center. Given an equation of a circle, they draw the graph in the coordinate plane, and apply techniques for solving quadratic equations, which relates back to work done in the first course, to determine intersections between lines and circles or parabola and between two circles.

Critical Area 6: Building on probability concepts that begin in the middle grades, students use the languages of set theory to expand their ability to compute and interpret theoretical and experimental probabilities for compound events, attending to mutually exclusive events, independent events, and conditional probability. Students should make use of geometric probability models wherever possible. They use probability to make informed decisions.
The assessments and strategies for learning listed on this page are not meant to be an “end all”. Additionally, these are repeated for every grade level curriculum guide with the intent that our students experience some of the same strategies year to year, teacher to teacher. On the following pages, you will find specific usage for some of these.

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<tr>
<th>Classroom Assessment For Learning</th>
<th>Literacy Connections</th>
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<tr>
<td>Assessment for learning provides students with insight to improve achievement and helps teachers diagnose and respond to student needs. (Stiggins, Rick, Judith Arter, Jan Chappuis, Steve Chappuis. Classroom Assessment for Student Learning. Educational Testing Service, 2006.)</td>
<td>Comprehensive literacy is the ability to use reading, writing, speaking, listening, viewing and technological skills and strategies to access and communicate information effectively inside and outside of the classroom and across content areas. (CLP, p.11)</td>
</tr>
</tbody>
</table>

Formative Assessment tools include:
- Constructed response items
- Descriptive Feedback (oral/written)
- Effective use of questioning
- Exit slips
- Milwaukee Math Partnership (MMP) CABS
- Observational checklists and anecdotal notes
- Portfolio items
- Projects (PBL)
- Student journals
- Student self-assessment
- Students analyze strong and weak work samples
- Use of Learning Intentions
- Use of rubrics with students
- Use of talk formats and talk moves, p. 55 of the CMSP
- Use of Success Criteria

Summative Assessment:
- Unit Tests
- WKCE/WAA

Progress Monitoring Tool
- Measure of Academic Progress (MAP)

*NOTE: The misconceptions listed within the template are only samples. You will encounter additional misconceptions as you work with students.*
Mathematics – High School Geometry Course

Standards for Mathematical Practice

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Modeling

Modeling Standards

Modeling is best interpreted in relation to other standards, not as a collection of isolated topics. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (*).

The basic modeling cycle is summarized in the diagram. It involves (1) identifying variables in the situation and selecting those that represent essential features, (2) formulating a model by creating and selecting geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between the variables, (3) analyzing and performing operations on the relationships to draw conclusions, (4) interpreting the results of the mathematics in terms of the original situation, (5) validating the conclusions by comparing them with the situations, and then either improving the model or, it is acceptable, (6) reporting on the conclusions and the reasoning behind them. Choices, assumptions, and approximations are present throughout this cycle.
# Mathematics – High School Geometry Course

## Experiment with transformations in the plane

1. Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

2. Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).

3. Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.

4. Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

5. Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

## Understand congruence in terms of rigid motions

6. Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.

7. Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.

## Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
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<table>
<thead>
<tr>
<th>Geometry</th>
<th>Congruence</th>
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<tbody>
<tr>
<td>G-CO</td>
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</table>

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<table>
<thead>
<tr>
<th>8. Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.</th>
</tr>
</thead>
</table>

**Prove geometric theorems**

9. Prove theorems about lines and angles. Theorems include: **vertical angles are congruent**; **when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent**; **points on a perpendicular bisector of a line segment are exactly those equidistant from the segment’s endpoints**.

10. Prove theorems about triangles. Theorems include: **measures of interior angles of a triangle sum to 180°**; **base angles of isosceles triangles are congruent**; **the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length**; **the medians of a triangle meet at a point**.

11. Prove theorems about parallelograms. Theorems include: **opposite sides are congruent**, **opposite angles are congruent**, **the diagonals of a parallelogram bisect each other**, and **conversely, rectangles are parallelograms with congruent diagonals**.

**Make geometric constructions**

12. Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.

13. Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.
Mathematics – High School Geometry Course

### Congruence

<table>
<thead>
<tr>
<th>Essential/Enduring Understanding</th>
<th>Assessments</th>
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<tbody>
<tr>
<td>Congruence is defined in terms of rigid motions: two objects or figures are congruent if there is a rigid motion that carries one onto the other</td>
<td>Students should be able to recognize and perform transformations using various geometric tools (i.e. patty paper, graph paper, etc.)</td>
</tr>
<tr>
<td></td>
<td>Students should be able to determine and define which rigid motion will move one given congruent figure to another</td>
</tr>
</tbody>
</table>
| | CABS (Classroom Assessments Based on Standards)  
  - Reflect This  
  - Draw Me  
  - Sketch It |

### Common Misconceptions/Challenges

- Congruence is defined in terms of rigid motions: two objects or figures are congruent if there is a rigid motion that carries one onto the other
- Students have a difficult time using the definitions. For example, when doing rotations, many students forget to identify the point and/or the angle of rotation.
- Students have a difficult time seeing the connections amongst exploratory results, conjectures, constructions, and formal proofs
- Students fail to understand that congruence theorems and postulate like ASA imply SSS and AAA

### Instructional Practices

**G-C: Congruence**

- Students develop an understanding of vocabulary by modeling with geometry: using patty paper or dynamic software to deepen understanding of congruency of objects.

**Example:** Students can model a rotation by tracing over a figure, then putting your pencil point on a point on the patty paper and rotating the patty paper about the point.
- Develop and test conjectures about the congruence of objects and/or figures by demonstration, constructions, counter examples, or dynamic software

  Example: Students may use dynamic software to demonstrate a sequence of transformations to show congruence of figures.

- Make and justify conjectures about congruent figures

  Example: Students may use dynamic software to demonstrate rigid motions to show the congruence postulates SSS, SAS, ASA, AAS, and Hypotenuse-Leg

**Differentiation**

Congruence is defined in terms of rigid motions: two objects or figures are congruent if there is a rigid motion that carries one onto the other

- Using a variety of representations can help make transformations more understandable to a wider range of students than can be accomplished by working with symbolic representations alone.
- Allow students to explore the congruency of two figures by using hands-on and visual experiences (tessellations)

**Literacy Connections**

- Use word walls to help students use the appropriate vocabulary when proving congruency and to build on prior knowledge
- Have students create a flow chart to better demonstrate relationships among the ideas in a proof

**Academic Vocabulary**

- Congruent
- Image
- Reflection
- Rigid Motion
- Rotation
- Translation
- Transformation
- Triangle Congruency

**Resources**

- Geometer’s Sketchpad
- Patty Paper
- Geometric tools
- For students to explore the path a line segment takes as it rotates around a distant point go to the applet at http://www.geometric-thinking.org/Rotation.htm
- Keypress.com (this site requires a log in)
Mathematics – High School Geometry Course

Standards for Mathematical Practice
1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning

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<tr>
<td><strong>Similarity, Right Triangles, and Trigonometry</strong></td>
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<tr>
<td><strong>G-SRT</strong></td>
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**Understand similarity in terms of similarity transformations**
1. Verify experimentally the properties of dilations given by a center and a scale factor:
   a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.
   b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.
2. Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.
3. Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.

**Prove theorems involving similarity**
4. Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.
5. Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

**Define trigonometric ratios and solve problems involving right triangles**
6. Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
7. Explain and use the relationship between the sine and cosine of complementary angles.
8. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.
Similarity, Trigonometry, and Geometric Practices

**Mathematics – High School Geometry Course**

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<thead>
<tr>
<th>Essential/Enduring Understanding</th>
<th>Assessments</th>
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<tbody>
<tr>
<td>▪ Similarity is defined as transformations</td>
<td>▪ Students can communicate logical arguments to show the truth of statements pertaining to the similarity of triangles</td>
</tr>
<tr>
<td>▪ Properties of similar triangles allow us to define the trigonometric ratios</td>
<td>▪ CABS</td>
</tr>
<tr>
<td>▪ Corresponding lengths of similar figures are proportional, but their angles are congruent</td>
<td>▪ Scott’s Reasoning (Grade 8)</td>
</tr>
</tbody>
</table>

**Common Misconceptions/Challenges**

**Similarity is defined as transformations**
- Students apply additive thinking to multiplicative situations. For example, students believe if you add the same number (such as 3) to the length of each side of a triangle the result will be a similar triangle. To address this misconception, have students construct, with patty paper, an original triangle, a triangle with three added to all sides, and a triangle with three times each side. Have students measure and compare angles and justify which triangle is similar to the original triangle.

**Properties of similar triangles allow us to define the trigonometric ratios**

**Corresponding lengths of similar figures are proportional, but their angles are congruent**
- Students believe that if two triangles are similar, the larger of the two triangles has larger angles. To address this misconception, have students trace the similar triangles on patty paper to match the corresponding angles to show congruency.

**Instructional Practices**

**G-SRT: Similarity, Right Triangles, and Trigonometry**
- Students develop an understanding of vocabulary by modeling with geometry: using patty paper, cubes (or manipulative), or geometric software to deepen understanding of similar objects

**Example:** Milo made a block structure with 6 blocks. Now he wants to make a structure with the same shape (i.e., a similar shape) and with triple the dimensions of the original. How many blocks will he need? Explain how you get your answer. Students can construct the figure and describe the resulting structure.

**Example:** When comparing two polygons, use patty paper to compare all corresponding angles and how the angles compare. Have students measure the corresponding segments of the polygons and find the ratio of the sides. How do the ratios of the corresponding sides compare?
- Allow students to create their own mural to understand similarity in real world application

  **Example**: Make a copy of a small drawing, cartoon, or picture. Using a pencil, lightly draw a grid of squares on the copy. Your mural will be more accurate if you use a grid of more small squares instead of fewer large squares. Then use a pencil to divide the mural surface into a similar but larger grid. Proceeding square by square, carefully draw the lines and curves of the drawing in the small squares into their corresponding large squares. Complete the mural by erasing the grid lines and coloring or painting the appropriate regions. Include an original drawing, a cartoon, or a photograph divided into a grid of squares. A finished mural can be drawn on a large sheet of paper.

- Have students show counterexamples to show polygons are similar

  **Example**: Remind students that a counterexample to a conjecture in the form “If \( P \), then \( Q \)” would show how \( P \) can be true but \( Q \) false. Students might recognize that the counterexamples make a case only for quadrilaterals. Challenge students to experiment with other polygons.

### Differentiation

**Similarity is defined as transformations**

- Have students use a variety of hands-on tools (i.e. manipulative, geometric software, java applets, etc) to experience a dilation of a figure. Some students may be able to understand similarity with graph paper, while others may need to use geometric tools.

**Properties of similar triangles allow us to define the trigonometric ratios**

  - Corresponding lengths of similar figures are proportional, but their angles are congruent

    - For students who need more help with exploration, break this task into steps for students to be able to follow. For example, when exploring AA as a shortcut to similarity, create step by step instruction for students. Some students may need a more open-ended questions like is AA a similarity shortcut?

### Literacy Connections

- Students use a journal to deepen their understandings of rigid and non-rigid transformations
- Students have to create a counterexample to show similarity

### Academic Vocabulary

- Dilation
- Scale Factor
- Triangle Similarity

### Resources

- Geometer’s Sketchpad
- Manipulative (cubes, patty paper, tangrams, pattern blocks, etc)
- Java applets
- Keypress.com (this site requires a log in)
Mathematics – High School Geometry Course

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<td>Circles</td>
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**Understand and apply theorems about circles**

1. Prove that all circles are similar.

2. Identify and describe relationships among inscribed angles, radii, and chords. *Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.*

3. Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.

4. (+) Construct a tangent line from a point outside a given circle to the circle.

**Find arc lengths and areas of sectors of circles**

5. Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.

**Standards for Mathematical Practice**

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### Circles

#### Essential/Enduring Understanding:
- All circles are similar
- Arc length of a circle is proportional to the radius and to the central angle defining the arc
- Area of a sector is proportional to the square of the radius and the central angle

#### Assessments
- Apply the definitions and properties of circles to solve problems
- Communicate logical arguments to show the truth of theorems pertaining to circles and explain clearly why the reasoning is valid

#### Common Misconceptions/Challenges

**All circles are similar**
- Students understand that as a “number” with no connection to the circle, not as a ratio of the circumference to the diameter of a circle. To address this misconception, have students measure the circumference and diameter of various objects, make a table of the results, and make a conjecture about the relationship between the circumference and diameter.

**Arc length of a circle is proportional to the radius and to the central angle defining the arc**

**Area of a sector is proportional to the square of the radius and the central angle**
- Students don’t connect the formulas to the previous work of fractions. For example, to find the area of a sector of a circle, students believe they need to remember another formula without exploring how the formula relates to the area of a sector, the area of a circle, and their previous work with fractions. To help students develop an understanding, have them derive the formulas of circles first before they apply the formula to a circle.

#### Instructional Practices

**G-C: Circles**
- Have students discover that is a ratio of the circumference to the diameter of a circle

  **Example:** Have students measure the circumference and diameter of various circular items. Have them chart their measurements then calculate the ratio of circumference to diameter for each circular item. Have students make a comparison about the relationship between the circumference and the diameter of the circle to make a conjecture.

- Have students investigate the properties of circles through constructions and forming their own conjectures
**Example:** Use what you know about inscribed angles and exterior angles of a triangle to find the missing angle measures in each diagram. Examine these cases to find a relationship between the measurement \( <AEN \) and the measures of the two intercepted arcs, \( \text{arc } \) and \( \text{arc } \). Complete the conjecture: The measure of an angle formed by two intersecting chords is \________________________\.

![Diagram of inscribed angles and exterior angles]

**Differentiation**

**All circles are similar**
- When exploring circles, use a variety of real-world examples of all circles where students may interact with, but also challenges students use their spatial reasoning as well as algebraic reasoning.

**Arc length of a circle is proportional to the radius and to the central angle defining the arc**
- Bring in a variety of circular objects to have students discover, circumference, and other properties of circles.

**Area of a sector is proportional to the square of the radius and the central angle**
- For an extension: Using geometry software, construct a circle and an arc on the circle. Measure only the circle’s circumference and the arc’s central angle. Calculate an expression for the arc’s length. Now measure the arc length to confirm that your calculation is correct. Write a paragraph explaining what you did.

**Literacy Connections**
- Students will be writing their own conjectures along with a labeled diagram throughout their journal and will communicate their reasoning

**Academic Vocabulary**
- Central Angle
- Congruent circles
- Concentric circles
- Chord
- Diameter

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<tr>
<td>Expressing Geometric Properties with Equations</td>
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</table>

G-GPE

Translate between the geometric description and the equation for a conic section
1. Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

Use coordinates to prove simple geometric theorems algebraically
4. Use coordinates to prove simple geometric theorems algebraically. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2).

5. Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).

6. Find the point on a directed line segment between two given points that partitions the segment in a given ratio.

7. Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.*

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expressing geometric properties with equations

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<th>essential/enduring understanding:</th>
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<tbody>
<tr>
<td>▪ pythagorean theorem in the guise of the distance formula can be used to derive equations of circles and parabolas, and other conic sections</td>
<td>▪ students can use geometry to model a real life problem in two- and three- dimensional situations</td>
</tr>
<tr>
<td>▪ a line can be determined by giving one point on the line and the direction of the line; on a coordinate plane, the direction of the line can be described by its slope</td>
<td>▪ cabs</td>
</tr>
<tr>
<td></td>
<td>▪ triangle pbj</td>
</tr>
<tr>
<td></td>
<td>▪ perpendicular bisector</td>
</tr>
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<td></td>
<td>▪ parallel lines</td>
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common misconceptions/challenges

| pythagorean theorem in the guise of the distance formula can be used to derive equations of circles and parabolas, and other conic sections |
|▪ students don’t see the connection between the equation of a circle and the definition of a circle. for example, when deriving the equation for a circle, surface that a circle is the set of all points in a plane at a given distance from a given point. therefore, applying the students understanding of the pythagorean theorem which is connected to the distance formula, which in turn is connected to the equation of a circle. |

| a line can be determined by giving one point on the line and the direction of the line; on a coordinate plane, the direction of the line can be described by its slope |
|▪ students have a difficult time visualizing where the points are without the grid lines present. for example, plotting and finding the distance or slope between two points when not given a coordinate grid. challenge students to graph points without having the grids and eventually allow them to find the distance and slope between two points. |

<table>
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<th>instructional practices</th>
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<tbody>
<tr>
<td>g-gpe: expressing geometric properties with equations</td>
</tr>
<tr>
<td>▪ have students investigate the use of a coordinate grid to apply the pythagorean theorem to derive the distance formula and then to further their understanding of the equation of a circle</td>
</tr>
</tbody>
</table>

| example: the distance formula is derived directly from the pythagorean theorem. students may benefit from being reminded that a circle is a collection of points equidistant from the center, where the equation of a circle is satisfied by the coordinates of a point if and only if the point lies on the figure. |
|▪ graph polygons, and then have students use their understanding of lines and algebraic reasoning to determine the shape. |
**Example**: Remind students that line segments can have distance, direction, and magnitude.

<table>
<thead>
<tr>
<th>Differentiation</th>
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<tr>
<td><strong>Pythagorean Theorem in the guise of the distance formula can be used to derive equations of circles and parabolas, and other conic sections</strong></td>
</tr>
<tr>
<td>- Have students use dynamic software to explore the definitions of the circles and parabolas.</td>
</tr>
</tbody>
</table>

A line can be determined by giving one point on the line and the direction of the line; on a coordinate plane, the direction of the line can be described by its slope

- Challenge students to graph points without having the grids and eventually allow them to find the distance and slope between two points. If students struggle with this transition, scaffold explorations. For example, without a coordinate grid, have student plot (2,3) using that as a reference, have them plot (3,4) and justify the placement of the point.

<table>
<thead>
<tr>
<th>Literacy Connections</th>
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<tbody>
<tr>
<td>- Students use a concept map to organize their thoughts in proving geometric concepts</td>
<td>- Circles</td>
</tr>
<tr>
<td>-</td>
<td>- Conic Section</td>
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<tr>
<td>-</td>
<td>- Distance Formula</td>
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<tr>
<td>-</td>
<td>- Parabolas</td>
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<tr>
<td>-</td>
<td>- Parallel</td>
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<td>- Perpendicular</td>
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<td>-</td>
<td>- Polygons</td>
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<td>-</td>
<td>- Pythagorean Theorem</td>
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<td>-</td>
<td>- Slope</td>
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<tr>
<td>- Geometer’s Sketchpad</td>
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<td>- Cabri Jr.</td>
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<td>- Keypress.com (this site requires a log in)</td>
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Mathematics – High School Geometry Course

Standards for Mathematical Practice
1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning

<table>
<thead>
<tr>
<th>Geometry</th>
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<tbody>
<tr>
<td>Geometric Measurement and Dimension</td>
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</tbody>
</table>

Explain volume formulas and use them to solve problems
1. Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri’s principle, and informal limit arguments.

2. (+) Give an informal argument, using Cavalieri’s principle for the formulas for the volume of a sphere and other solid figures.

3. Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.*

Visualize relationships between two-dimensional and three-dimensional objects
4. Identify the shapes of two-dimensional cross-sections of three dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.
### Mathematics – High School Geometry Course

#### Geometric Measurement and Dimension

<table>
<thead>
<tr>
<th>Essential/Enduring Understanding</th>
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<tr>
<td>If two figures have the same height and the same cross sectional area at every level, then they have the same volume (Cavalieri’s Principle)</td>
<td>Have students create and present a demonstration of Cavalieri’s Principle</td>
</tr>
<tr>
<td>Cavalieri’s principle can be used to derive volume formulas for cylinders and cones.</td>
<td>CABS</td>
</tr>
<tr>
<td></td>
<td><em>Chubby Checkers</em></td>
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<td></td>
<td><em>Tommy’s Deck</em></td>
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</tbody>
</table>

#### Common Misconceptions/Challenges

If two figures have the same height and the same cross sectional area at every level, then they have the same volume (Cavalieri’s Principle) Cavalieri’s principle can be used to derive volume formulas for cylinders and cones.

- Some students might believe that if two solid have the same volume, then they are the same shape. For example, when given an oblique prism and a regular prism with the same height and cross-sectional area, ask students to make conjectures about their volumes. (See pictures in the Instructional Practices) If you surfaced this misconception, push students on this idea throughout the exploration of Cavalieri’s Principle.

#### Instructional Practices

**G-GMD: Geometric Measurements and Dimension**

- Use physical models to demonstrate Cavalieri’s Principle

*Example:* You can approximate the shape of this oblique rectangular prism with a staggered stack of three reams of 8.5-by-11-inch paper. If you nudge the individual pieces of paper into a slanted stack, then your approximation can be even better. Rearranging the paper into a right rectangular prism changes the shape, but certainly the volume of paper hasn’t changed. The area of the base, 8.5 by 11 inches, didn’t change the height, 6 inches, didn’t change, either. In the same way, you can use crackers, CDs, or coins to show that an oblique cylinder has the same volume as a right cylinder with the same base and height. See diagram.
Make connections to the *Moving and Combining principles* from previous grades (6-8) in the common core state standards

**Example**: Have students derive the area of a parallelogram by rearranging familiar shapes. Students construct a parallelogram, from the vertex of the obtuse angle, adjacent to the base, and draw an altitude to the side opposite the base. Cut out the parallelogram and then cut along the altitude. Arrange the two pieces into other shapes without overlapping them. See diagram below that shows the moving and combining principle.

![Diagram of a parallelogram](image)

**Differentiation**

*If two figures have the same height and the same cross sectional area at every level, then they have the same volume (Cavalieri’s Principle)*

Cavalieri’s principle can be used to derive volume formulas for cylinders and cones.

- Have students use “unit cubes” to understand volume.
- Use a variety of physical models to demonstrate Cavalieri’s Principle.

**Literacy Connections**

<table>
<thead>
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*Math Curriculum Guide Geometry_07.28.11_v1*

This Curriculum Guide is to be used in conjunction with the Pacing Guide and Instructional Design Lesson Plan. The Instructional Design will help with lesson planning using Launch-Explore-Summarize.
- Students will be using “speaking” to present a demonstration of Cavalieri’s Principle

- Altitude
- Cavalieri’s Principle
- Cone
- Cylinder
- Edge
- Face
- Polyhedron names
- Vertex
- Volume

**Resources**

- Various websites to explore Cavalieri’s Principle: illuminations.nctm.org, mathworld.wolfram.com and cut-the-knot.org
- Keypress.com (this site requires a log in)
- Geometer’s Sketchpad
Mathematics – High School Geometry Course

Standards for Mathematical Practice
1. Make sense of problems and persevere in solving them.
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### Geometry

#### Modeling with Geometry *

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<tr>
<td><strong>Geometry</strong></td>
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<td><strong>Modeling with Geometry</strong></td>
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**Apply geometric concepts in modeling situations**

1. Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder). *

2. Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot). *

3. Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios). *
# Mathematics – High School Geometry Course

## Modeling with Geometry

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<tr>
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<tbody>
<tr>
<td>▪ Geometric models can be used to represent and analyze real world situations</td>
<td>▪ Create a presentation of a solution to a real world problem using geometric models.</td>
</tr>
</tbody>
</table>
| ▪ | ▪ CABS  
| | ▪ *Chubby Checkers*  
| | ▪ *What Do You Know?*

### Common Misconceptions/Challenges

**Geometric models can be used to represent and analyze real world situations**

▪ Students have a difficult time understanding the relationship between two- and three-dimensional shapes when applied to a contextual situation. For example, students don’t see the connection of a circle as a model for the approximate cross-section of an elephant’s leg. To address this challenge, bring in physical models from the science lab for students to explore.

### Instructional Practices

**G-MG: Modeling with Geometry**

▪ The teacher needs to create situations where students are expected to use modeling to represent and analyze the real world. To complete the modeling cycle, students need to reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

*Example:* Explore the relationship between size and function in living creatures. Imagine two similar animals, one with dimensions three times as large as those of the other. How would the surface area of these two animals compare? How would the volume of these two animals compare? (See Resources from Key Curriculum)

### Differentiation

**Geometric models can be used to represent and analyze real world situations**

▪ To help students visualize word problems (i.e. what is the longest fishing pole that can fit in a box 7ft by 2ft by 1ft), bring in physical models to allow students to demonstrate the situation. Allow students to explore how a fishing pole could fit into a box (ex: What are some ways a fishing pole could fit into a rectangular prism (box)).
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<tr>
<td>▪ Students will use their “speaking” skills to present a real world problem using geometric models</td>
<td>▪ Analyze</td>
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<td>▪</td>
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<tr>
<td>▪ The enrichment resources in Key Curriculum Discovering Geometry, More Projects and Explorations, to deepen students’ contextual understanding through modeling.</td>
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