THEORETICAL PREDICTIONS OF ELASTIC PROPERTIES OF CEMENTITIOUS COMPOSITES MADE WITH AND WITHOUT MINERAL ADMIXTURES

By Tarun R. Naik and Shiw S. Singh

Report No. 161
June, 1992

Department of Civil Engineering and Mechanics
College of Engineering and Applied Science
THE UNIVERSITY OF WISCONSIN - MILWAUKEE
THEORETICAL PREDICTIONS OF ELASTIC PROPERTIES OF CEMENTITIOUS COMPOSITES MADE WITH AND WITHOUT MINERAL ADMIXTURE

By

Tarun R. Naik
Director, Center for By-Products Utilization

and

Shiw S. Singh
Post-Doctoral Fellow
Center for By-Products Utilization

Department of Civil Engineering and Mechanics
College of Engineering and Applied Science
The University of Wisconsin-Milwaukee
P.O. Box 784
Milwaukee, WI  53201
Telephone: (414) 229-6696
Fax: (414) 229-6958
Theoretical Predictions of Elastic Properties of Cementitious Composites Made With or Without Mineral Admixtures

By

Tarun R. Naik and Shiw S. Singh

ABSTRACT

This research work was carried out to establish an appropriate theoretical model to describe elastic properties of cementitious composites. The elastic properties include modulus of elasticity, modulus of rigidity, bulk modulus and Poisson's ratio. A literature search was carried out to gain information about existing analytical models in order to predict elastic behavior of composites. Analysis of the literature information collected revealed that mechanical behavior of concrete can be modelled by using available models for composites.

In this work, plain concrete was considered as a particulate composite in which matrix (hardened paste) is made up of cement, sand, and ash, and reinforcement is represented by coarse aggregates particles. In order to evaluate validity of theoretical models, elastic properties of concrete specimens were also determined experimentally for fly ash concrete proportioned to have 28-day compressive strength of 6000 psi (41 MPa) using ASTM Class F fly ash from two different sources. Fly ash content was varied between 40 to 60 percent. The result of this investigation has established an excellent model for representing elastic properties of fly ash concretes.

Keywords:Elastic properties; composite; theoretical model; modulus of elasticity; modulus of rigidity; bulk modulus; fly ash.

INTRODUCTION

A composite material is composed of two or more materials (homogeneous phases) differing in composition or form. One of these phases is continuous solid phase, generally called matrix. The discontinuous phases are reinforcing phases which may be in either a fiber or particle form. Load carrying capacity of a composite material depends greatly upon the properties of participating constituents, volume fraction, geometry, size, orientation, spatial distribution of the reinforcing fillers, and interaction that can occur between the reinforcements, and between the reinforcements and matrix.

Concrete is a multiphase material. In this investigation, a plain concrete was considered for modeling purposes. A plain concrete is a particulate composite in which matrix is made of cement, sand and/or ash, and reinforcing filler is represented by coarse aggregates. Both the matrix and aggregate phases are also composite materials. The properties of these can be separately modeled by using theories of composites.
The matrix is a multiphase material which can be composed of a binder formed from the hydration of cement (a mixture of cement and water), sand and ash or other fine aggregates, and voids present in the system. Therefore, matrix properties can be altered by varying volume fraction of these constituents. Though they are heterogenous materials on a microscopic scale, they can be approximated by homogeneous materials on a macroscopic level whose properties remain uniform. Similarly, each aggregate particle on microscopic level is also a multiphase material. Aggregate particles are made up of several minerals and contain voids. In fact, concrete is a complex hybrid composite whose properties can vary greatly from one point to another due to variation in its structure. The weakest region is the transition zone which is the interfacial region between the matrix and the coarse aggregate particles. This zone consist of solids and is more porous compared to the other regions of the matrix.

On a macroscopic level, concrete can be treated as a two-phase composite consisting of the mortar matrix and reinforcing filler as coarse aggregate. Additionally, void can also be considered as the third phase in modeling particulate composites like plain concretes.

EFFECTIVE LENGTH OF REINFORCING FILLER MATERIALS

The performance of the filler reinforcement in a composite depends heavily upon several factors including its shape, size, geometry, amount, material strength properties, etc. of the fiber. The length of the fiber affects load resisting capability of a composite to a great extent.

Applied load to a composite is transmitted to the fiber by the matrix. During the loading, the fiber and matrix experience different displacement due to variations in their modulus values. As a result, shear strains at the interface between the matrix and the fiber are generated in a direction parallel to the fiber axis. Thus, the matrix transmits the applied load to the fiber due to the shear strain generated.

Considering an end segment of the fiber of length x, as shown in Figure 1,
FIGURE 1: A COMPOSITE MODULUS SHOWING STRESS TRANSMISSION TO A FIBER [2].
equilibrium of forces can be written as [1]:

\[ \sigma_x r^2 = \int_0^x \tau(x) 2\pi r dx + \sigma_m \pi r^2 \]  

Where

- \( \sigma_x \) = tensile stress in fiber applied,
- \( \tau(x) \) = shear stress generated at the interface as a function of \( x \),
- \( r \) = radius of the fiber, and
- \( \sigma_m \) = tensile stress in the matrix.

For a larger value of \( x \), the second term in Eq. 1 can be ignored. Eq. 1 indicates that the fiber stress will grow with an increase in length until the fiber attains its ultimate strength. Hence, there is a critical length (\( l_c \)) at which the stress transmitted to the fiber approaches its ultimate stress (\( \sigma_{fu} \)) value while matrix deforms plastically around the interface. This means that when \( x \) becomes equal to \( l_c \), then \( \tau \) becomes equal to matrix yield strength \( \tau_y \). For such a condition, neglecting tensile stress transmission to the fiber end, Eq. 1 can be presented as:

\[ \frac{1}{2} \sigma_{fu} \frac{l_c d}{4} = \frac{1}{2} \sigma_{fy} \frac{l_c d}{4} \]

Where \( d \) is diameter of the fiber. The effect of fiber length on stress transmission to fiber is presented in Figure 2. When the length of the fiber (\( l \)) equals to \( l_c \), the effective portion of the reinforcing composite is approximately half of the real fraction as area of the triangle abc is nearly half of the value \( \sigma_{t} l_c \) (Figure 2).

From the above description, it appears that strength, especially tensile strength, will decrease as a result of the addition of particle fillers to a matrix. However, in the above relations the stress transmission to a particle ends has been neglected. Therefore, generally, addition of particles in small quantities does not decrease strength of the matrix.

The negative effects of particulate fillers on the strength of the composite, can be related to stress concentration in the vicinity of the particles, and partial transmission of stresses from the matrix to filler. This results in a decrease in strength of composites [1-4], see Figures 3 and 4. The positive effects of particulate reinforcements are increases in elastic modulus of the matrix (Figure 5), hardness (Figure 6), etc.

In the case of a compressive load, the effect of defects, void, and poor bond between particles and matrix is concealed to a marked extent. Therefore, addition of rigid particle reinforcements generally increases strength of composites up to a certain level (Figure 7). The same is true for concrete filled with large aggregates (relatively rigid filler particles).
MODELS FOR COMPOSITES REINFORCED WITH PARTICULATES

A simplest form of model known as rule of mixture (ROM) is used to obtain approximate properties of composites [1,2]. The ROM predicts properties of composites based on weighted volume-average of participating constituents. For example the ROM for modulus of elasticity can be expressed as [1]:

\[ E_c = V_m E_m + V_p E_p \]  \hspace{1cm} (5)

where \( E \) is modulus of elasticity and \( V \) is volume fraction. The subscripts \( c, m, \) and \( p \) refers to the composite, matrix and the reinforcing filler.

The theory of linear elasticity is used in developing theoretical models for representing elastic properties of composites by a number of researchers [6-15]. All these models involve simplifying assumptions applied to the Theory of Elasticity. The detailed descriptions of these models are given in the references cited above. Some of the theoretical models suitable for concrete composites are selected for the study reported herein.

For small concentration of filler particles, Hashin [8] developed a relation for shear modulus of composite as given by Eq. 6:

\[ \frac{G_c}{G_m} = \left[ 1 - 15 (1 - \mu_m) (1 - \frac{G_p}{G_m}) V_p \right] [1 - 5 \mu_m - 2 (4 - 5 \mu_m) (\frac{G_c}{G_m})] \]  \hspace{1cm} (6)

where:
- \( G \) = Modulus of rigidity or shear modulus
- \( V \) = Volume fraction
- \( \mu \) = Poisson's ratio
- \( c, m, \) and \( p \) = indices for composite, matrix, and particle, respectively.

The relation of Eq. 6 was first established by Dewey [7]. In the case of small concentration of filler materials, particle to particle interactions are assumed to be negligible.

For the special case when the particle is perfectly rigid and the matrix is incompressible (\( \mu = 1/2 \)), the Eq. 6 is reduced to Eq. 7.

\[ \frac{G_c}{G_m} = (1 + \frac{5}{2} V_p) \]  \hspace{1cm} (7)

The model for Bulk Modulus for small concentration of filler material as derived by Hashin [8] is given by Eq. 8.
where \( K \) is the bulk modulus, and all other symbols have the same meaning as described before.

After knowing shear and bulk moduli of composites, Young's modulus can be computed as:

\[
E_c = \frac{9K_cG_c}{(3K_c+G_c)}
\] (9)

Poisson's ratio, for known \( K_c \) and \( G_c \) can be estimated from relation of the form given by Eq. 10.
FIGURE 2: EFFECT OF FIBER LENGTH \((l)\) ON TENSILE STRESS TRANSMISSION FROM MATRIX TO FIBER \([1,2]\).
FIGURE 3: NORMALIZED ULTIMATE TENSILE STRENGTH UTS VS. VOLUME FRACTION OF PARTICLES FOR COMPOSITES [4].
FIGURE 4: VARIATION OF YIELD STRENGTH AND ULTIMATE TENSILE STRENGTH OF ASHALLOY WITH ASH CONTENT [5].
FIGURE 5: VARIATION OF ELASTIC MODULUS OF ASHALLOY WITH ASH CONTENT [5].
FIGURE 6: PLOT SHOWING INCREASE IN HARDNESS OF ASHALLOY WITH FLY ASH CONTENT [5].
FIGURE 7: 0.5\% COMPRESSIVE PROOF STRENGTH FOR DIFFERENT ASHALLOYS AND MATRIX ALLOYS [5].
For composites containing large volumes of filler particles, particle to particle interactions would become significant. This requires a knowledge of particle size distribution and arrangement in order to develop theoretical models. Due to complexities involved for high filler concentrations, exact general solutions have not yet been obtained [1,6]. Various models are developed by making some simplifying assumptions, including small deformation and linear elastic behavior of the material governed by Theory of Elasticity.

A composite sphere model (polydisperse model) was proposed by Hashin [1,6]. In this model it is assumed that each spherical particle of a radius "a" is surrounded by a matrix layer of radius "b". The ratio of radii a/b is taken to be constant, irrespective of size of reinforcing particles, and the complete space is filled by the two-layer particles. This model demands a wide range of particle size distribution in order to maintain the ratio a/b constant while achieving a volume filling requirements (Figure 8). For high-volume fractions of particle fillers, the effective modulus of composite for the polydisperse was established [8] as given by Eq. 11.

\[ K_c - K_m / K_p - K_m = [V_p / [1 + (1-V_p)(K_p - K_m)/(4K_m + 3G_m)]] \]  \hspace{1cm} (11)

The composite model does not provide a general solution for modulus of rigidity of composites for all range of particle concentrations. For low-volume concentration of filler, the shear modulus relation is the same as given in Eq. 6. For the case of high-volume concentrations of fillers, the relation given by Eq. 12 holds [1,6].

\[ \frac{G_c}{G_p} = 1 - \left[ (1-G_m/G_p) (1-V_p) \right] \frac{7-5\mu_m+2(4+5\mu_m)G_p}{G_m} / [15(1-\mu_m)] \]  \hspace{1cm} (12)

A three-phase model was proposed by Kernér [9] and Van der Pool [10]. This model is composed of three phases, namely, matrix phase, spherical particle, and equivalent phase. Each spherical particle is surrounded by a matrix phase and this composite sphere is surrounded by an equivalent homogeneous media (Figure 9). The equivalent phase has unknown properties of composite (K_c and G_c). This model predicts the bulk modulus relation which is the same as that given by Eq. 6. A final solution for shear modulus was obtained in terms of a quadratic equation of the form [1,6] given by Eq. 13.

\[ A (\frac{G_c}{G_m})^2 + 2B (\frac{G_c}{G_m}) + C = 0 \]  \hspace{1cm} (13)

In Eq. 13, the values of A, B, and C are expressed in terms of shear modulus, Poisson's ratio, and volume fraction of particles and matrix. The exact relation between these parameters are given in the literature [1,6].
Paul [11] was the first to develop relation for lower and upper bounds for a two-phase composites. The former is a model composed of series elements whereas the latter is a model composed of parallel elements for a multi-phase material (Figure 10). For a two-phase composite system, the lower and upper bound for elastic modulus are represented by the relation of the form [11] given by Eq. 14 and 15.

\[ E_c(\text{lower}) = \frac{E_m E_p}{V_mE_p + V_p E_m} \]  \hspace{1cm} (14)
\[ E_c(\text{upper}) = V_mE_m + V_p E_p \]  \hspace{1cm} (15)
FIGURE 8: COMPOSITE SPHERES MODEL [6]
Unfortunately the bounds predicted by Eq. 14 and 15 are far apart for real composite systems. Therefore, efforts have been made by some researchers to develop closer bounds for representing elastic properties of composites [6, 12, 13].

Hashin and Shtrikman [12] established upper and lower bounds for shear modulus as given by Eq. 13 and 14.

\[
G_c(\text{lower}) = G_m + \frac{V_p}{6 \left( K_m + 2 G_m \right) V_m} \left( G_p - G_m \right) + \frac{1}{3 \left( 3 K_m + 4 G_m \right) G_m} \left( G_p - G_m \right)
\]

\[
G_c(\text{upper}) = G_p + \frac{V_m}{6 \left( K_p + 2 G_p \right) V_p} \left( G_p - G_m \right) + \frac{1}{3 \left( 3 K_p + 4 G_p \right) G_p} \left( G_p - G_m \right)
\]

The lower and upper bounds for bulk modulus obtained by Hashin and Shtrikman [12] are given by Eq. 18 and 19.

\[
K(\text{lower}) = K_m + \frac{V_p}{3 V_m} \left( K_p - K_m \right) + \frac{1}{3 \left( 3 K_m + 4 G_m \right)}
\]

\[
K(\text{upper}) = K_p + \frac{V_m}{3 V_p} \left( K_p - K_m \right) + \frac{1}{3 \left( 3 K_p + 4 G_p \right)}
\]

Hansen [14] modified the relation of bulk modulus derived by Hashin [8] in order to develop elastic modulus of concrete by assuming \( \mu_p = \mu_m = 0.2 \). The final form of the model is given by Eq. 20.

\[
E = \frac{(1 - V_p) E_m + (1 + E_p) E_p}{(1 + V_p) E_m + (1 - V_p) E_p}
\]

\( E \) =
FIGURE 9: A THREE PHASE MODEL [6]
FIGURE 10: UNIDIRECTION SERIES AND PARALLEL ELEMENTS OF TWO PHASE COMPOSITES.
It is well established that elastic properties of solid is adversely affected by presence of pores or voids in the materials [15-20]. MacKenzie [15] was probably the first to propose theoretical models for the shear and bulk modulus for solids containing spherical holes. His models for shear and bulk modulus were modified to express the relations of the form given by Eq. 21 and 22.

\[ G_o = G \left( 1 - \frac{4K(1+\mu)K}{4G+3K}V_{vo} \right) \]

where \( G_o \) and \( K_o \) are shear modulus and bulk modulus of porous solids, and \( G \) and \( K \) refers to shear and bulk modulus of non-porous solid; and, \( V_{vo} \) volume fraction of voids present in the solid.

A large number of empirical models are available for describing elastic modulus of solids. Most widely used models are polynomial in form, containing one or more material constants [16,17,18]. A generalized form is given by Eq. 23.

\[ E = E_o \exp \left[ - (aP + bP^2 + cP^3 + \ldots) \right] \]

where \( E \) and \( E_o \) are the elastic moduli at porosity \( P \) and zero, and \( a, b, c, \ldots \) are material constants. Several forms of models representing modulus relation with porosity is summarized by Phani and Niyogi [17].

It has been shown [20] that strength of a material as a function of porosity can be modelled as given by Eq. 24.

\[ \sigma = \sigma_o e^{-aP} \]

where \( \sigma \) is strength of material at a porosity \( P \), \( \sigma_o \) is strength of material at zero porosity, and \( a \) is a material constant.

EXPERIMENTAL INVESTIGATION

In order to judge validity of theoretical models, it was decided to obtain experimental data on fly ash concrete. In this study, concretes containing varying amounts of fly ash to have cement replacement in the range of 0-60% were used. A reference concrete was proportioned to have 28-day compressive strength of 6000 psi (41 MPa). Details of mixtures design have been reported elsewhere [21]. Experiments were performed to obtain elastic properties of concrete such as compressive strength, splitting tensile strength, and modulus of elasticity. These properties were measured using 6x12 in. (150 x 300 mm) cylinders in accordance with standard ASTM test methods.

DETERMINATION OF ELASTIC PROPERTIES BY THEORETICAL MODELS

In order to compute properties of concrete, it was assumed that all participating constituents
are elastic in nature, and concrete composite made with these materials is also elastic in nature. It is further assumed that each participating phase is homogeneous and isotropic.

The above assumptions permitted the use of elastic equations developed by the use Theory of Elasticity in predicting concrete properties, Eq. 6 through 19.

The concrete was modelled as a two-phase composite consisting of the coarse aggregate as particulate filler and cement matrix containing fly ash as continuous phase.

Density of concrete composite decreases after addition of fly ash. This can result in increase in porosity due to some hollow fly ash particles and poor bond between cement matrix and fly ash particles. The volume fraction of porosity can be approximated based on density measurement. In order to account for the stress concentration resulting due to increased porosity, the relation developed by McKenzie [15] for spherical holes was used to correct the values of concrete bulk and shear moduli. The final values of moduli predicted by theoretical models were used to determine Young's modulus and Poisson's ratio.

The models for composites require input parameters such volume fraction, shear modulus, bulk modulus and Poisson's ratio of participating constituents. In this investigation, Young's modulus of filler particles (natural gravel) was taken as 8 x 10^6 psi, and Poisson's ratios of particle fillers and matrix materials were taken as 0.15 and 0.20, respectively. The variable parameter is Young's modulus of matrix that would depend upon design strength of concretes. For 3000, 4000, 5000, and 6000 psi concretes, Young's modulus of matrix was taken as 2 x 10^6, 2.5 x 10^6, 3.5 x 10^6, and 4 x 10^6 psi, respectively. Steps involved in determination of theoretical values of elastic properties of plain concrete are as follows:

1) From known values of Young's modulus of matrix and filler, their shear and bulk moduli were computed by Eq. 25-28.

\[
\begin{align*}
G_p &= \frac{E_p}{2 (1+\mu_p)} \\
K_p &= \frac{E_p}{3 (1-2\mu_p)} \\
G_m &= \frac{E_m}{2 (1+\mu_m)} \\
K_m &= \frac{E_m}{3 (1-2\mu_m)}
\end{align*}
\] (25-28)

2) Volume fractions of the participating constituents were evaluated as by Eq. 29-32.
where

\[
VT = \frac{WCM}{DCM} + 0.22 \frac{WCM}{DW} + \frac{WFA}{DFA} + \frac{WAG}{DAG}
\]

\[
V_{vm} = \frac{WCM}{DCM} + \frac{0.22 WCM}{DW} + \frac{WFA}{DFA}
\]

\[
V_m = V_{vm} \times VT
\]

\[
V_p = \left( \frac{WAG}{DAG} \right) \times VT
\]

(3) After determining volume fractions of the constituents, matrix and filler particles, bulk and shear moduli of concrete composites can be determined by the composite models discussed previously (Eq. 6 through 19). A computer program was developed to check suitability of the various models for evaluating shear and bulk moduli under a wide range of input conditions. Based on the result derived, it was found that Eq. 6, 16 and 17 for shear modulus, and Eq. 8, 11, 18, and 19 for bulk modulus were adequate for representing concrete moduli.

(4) The shear and bulk moduli were corrected for porosity effects by Eq. 21 and 22 respectively, for concrete containing fly ash. From experimental density data, the values of volume fraction of voids was computed. The void relationship was expressed in terms of fly ash percentage by regression model given by Eq. 33.

\[
V_{vo} = 8.05 \times 10^{-4} FAP - 7.2 \times 10^{-7} FAP^2
\]
where 
\[ V_{vo} = \text{volume fraction of voids present in the concrete relative to no-fly ash concrete, assuming negligible porosity of plain portland cement concrete.} \]
\[ \text{FAP} = \text{percent fly inclusion} \]

(5) The values of elastic modulus was determined by Eq. 9, and Poisson's ratio by Eq. 10 from the known values of shear and bulk moduli from the step (4).

Concrete strength properties, especially elastic modulus are found to increase with addition of Class C fly ash up to approximately 30% cement replacement at the 28-day age. For such a case, concrete matrix can also be modelled as a two-phase system taking cement paste as a continuous phase and fly ash particles as a reinforcement. Then composite paste properties can be determined following the above steps 1, 2, 3, and 5. For determination of overall fly ash concrete properties, the resulting composite cement-fly ash matrix can be considered as a continuous phase and coarse aggregate particles can be considered as a reinforcing filler. Using this assumption, fly ash concrete properties can be determined by using the appropriate properties of the composite fly-ash matrix and coarse aggregate properties, and repeating the above steps 1, 2, 3, and 5. The same procedure can be adopted in determining elastic properties of concrete made with silica fume because silica fume also increases moduli of concrete. When high volumes of Class C fly as are used in concrete, the particle to particle interactions become significant which can cause decrease in the modulus values. Therefore, a correction similar to one described in step 4 for Class F fly as concrete can be used for the case of cement replacements with Class C fly ash beyond 30% level. Elastic properties of Class C fly ash concrete were also determined. The results are based on elastic modulus of fly ash particles of 14.6 x 10^6 psi and Poisson's ratio of 0.12. The elastic modulus value was estimated from known elastic moduli of Al_2O_3 and fused silica, and assumed value of other constituents of fly ash.

**RESULTS AND DISCUSSION**

Some trial results for concrete containing Class F fly ash are presented in Tables 1 through 4. Analytical results were obtained for concrete proportioned to have the 28-day compressive strengths of 3000, 4000, 5000, and 6000 psi, whereas experimental results were obtained for 6000 psi concretes containing high volumes of Class F fly ashes, Fig. 11. Theoretical determination of elastic properties of Class C fly ash concrete at 30% cement replacement was also done (Figure 12).

The Shear and bulk moduli were determined by the models proposed by Hashin and Shtrikman [12] as they showed the best results. Shear modulus of concrete was taken as the average of upper and lower bounds predicted by Eq. 16 and 17, respectively. Similarly, bulk modulus was represented by the average value of upper and lower bounds computed by Eq. 18 and 19. When Shear modulus was computed by Eq. 6 and bulk modulus by Eq. 8, the concrete moduli were slightly higher compared to prediction made by the above models. However, these models are also adequate to obtain concrete Shear and bulk moduli.

In this study, concrete properties were determined by models which were developed based
upon theory of linear elasticity. Concrete, like any other materials, can behave elastically for very small strains. However for finite strains, the behavior becomes non-linear. Therefore, discrepancies between theoretical and experimental values are bound to occur. However, in spite of several simplifying assumptions having been made in development of theoretical models for composites, the model predictions for concrete are encouraging.

Through the use of theoretical models, various properties of concrete composites such as Shear modulus, bulk modulus, Young's modulus, and Poisson's ratio were evaluated (Tables 1 through 4). Analysis of the results obtained showed that the differences between theoretical and experimental data were within the range of experiment error that can occur due to variations in concrete samples resulting from differences in sample preparations and testing. Modulus of elasticity of concrete was also computed by the ACI 318-89 code equation. The computed and experimental values of modulus of elasticity of concrete are compared in Figure 11.

In general, theoretical model for stress-strain relationship are less accurate compared to elastic moduli of materials. Therefore, in this study only theoretical models for the elastic moduli were selected and their validity was examined. The design compressive strength can be obtained by using empirical relations. Approximate value of compressive strength of concrete can be taken as $10^3$ times the Young's modulus value for design purposes.

Both theoretical and experimental models showed the same general trend regarding elastic behavior of concrete. As expected, theoretical values of concrete shear modulus, bulk modulus, Young's modulus increased with strength and decreased due to addition of Class F fly ash. Concrete Poisson's ratio decreased with increasing strength, and increased with increasing fly ash content.

The theoretically predicted elastic modulus values were close to the experimental values obtained in the present investigation, see Figure 11 [21]. Also, model predictions for Poisson's ratio were close to experimental values reported in the literature [22]. These results pointed out that the theoretical models for shear and bulk modulus proposed by Hashin and Shtrikman are sufficiently adequate for predicting elastic properties of structural grade concretes.

**CONCLUSIONS**

Based on the results derived in this investigation, it can be concluded that elastic properties of cementitious composites can be adequately described by the models developed for composite materials based upon linear theory of elasticity.

The models for shear and bulk moduli derived by Hashin and Shtrikman showed the best representation of experimental data. Therefore, it is recommended to use this model for elastic properties determination of concrete made with or without fly ash. If required, the model predictions can be corrected for porosity effects when Class F fly ash is added to the system by using McKenzie
FIGURE 11: A COMPARISON BETWEEN EXPERIMENTAL AND THEORETICAL MODEL PREDICTION OF MODULUS OF ELASTICITY OF FLY ASH CONCRETE
FIGURE 12: ELASTIC PROPERTIES OF 6000 PSI CLASS C FLY ASH CONCRETE AT 30% CEMENT REPLACEMENT
model in obtaining concrete elastic properties. The difference between theoretical and experimental values observed in this work were in the range experimental errors observed in measurement/testing of concrete.

REFERENCES


REP-161
### TABLE 1: ELASTIC PROPERTIES OF 3000 PSI NON-AIR ENTRAINED CONCRETE

<table>
<thead>
<tr>
<th>Fly Ash Content</th>
<th>Theoretical</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$G_c$ $10^6$ x psi</td>
</tr>
<tr>
<td>0</td>
<td>1.57</td>
</tr>
<tr>
<td>10</td>
<td>1.54</td>
</tr>
<tr>
<td>20</td>
<td>1.51</td>
</tr>
<tr>
<td>30</td>
<td>1.48</td>
</tr>
<tr>
<td>40</td>
<td>1.45</td>
</tr>
<tr>
<td>50</td>
<td>1.43</td>
</tr>
<tr>
<td>60</td>
<td>1.40</td>
</tr>
</tbody>
</table>

### TABLE 2: ELASTIC PROPERTIES OF 4000 PSI NON-AIR ENTRAINED CONCRETE

<table>
<thead>
<tr>
<th>Fly Ash Content</th>
<th>Theoretical*</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$G_c$ $10^6$ x psi</td>
</tr>
<tr>
<td>0</td>
<td>1.77</td>
</tr>
<tr>
<td>10</td>
<td>1.73</td>
</tr>
<tr>
<td>20</td>
<td>1.70</td>
</tr>
<tr>
<td>30</td>
<td>1.67</td>
</tr>
<tr>
<td>40</td>
<td>1.64</td>
</tr>
<tr>
<td>50</td>
<td>1.61</td>
</tr>
<tr>
<td>60</td>
<td>1.57</td>
</tr>
</tbody>
</table>

* $G_c$, $K_c$, $E_c$, and $\mu_c$ are shear modulus, bulk modulus,
Young's modulus and Poisson's ratio, respectively.
<table>
<thead>
<tr>
<th>Fly Ash Content</th>
<th>Theoretical</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$G_c \times 10^6$ psi</td>
</tr>
<tr>
<td>0</td>
<td>2.14</td>
</tr>
<tr>
<td>10</td>
<td>2.09</td>
</tr>
<tr>
<td>20</td>
<td>2.05</td>
</tr>
<tr>
<td>30</td>
<td>2.01</td>
</tr>
<tr>
<td>40</td>
<td>1.97</td>
</tr>
<tr>
<td>50</td>
<td>1.92</td>
</tr>
<tr>
<td>60</td>
<td>1.88</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Fly Ash Content</th>
<th>Theoretical*</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$G_c \times 10^6$ psi</td>
</tr>
<tr>
<td>0</td>
<td>2.30</td>
</tr>
<tr>
<td>10</td>
<td>2.25</td>
</tr>
<tr>
<td>20</td>
<td>2.21</td>
</tr>
<tr>
<td>30</td>
<td>2.16</td>
</tr>
<tr>
<td>40</td>
<td>2.11</td>
</tr>
<tr>
<td>50</td>
<td>2.07</td>
</tr>
<tr>
<td>60</td>
<td>2.02</td>
</tr>
</tbody>
</table>

* $G_c$, $K_c$, $E_c$, and $\mu_c$ are shear modulus, bulk modulus, Young's modulus and Poisson's ratio, respectively.
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Composite system</th>
<th>Matrix Composition</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Al-graphite</td>
<td>Al-11.8wt.%Si-1.5wt.%Cu</td>
</tr>
<tr>
<td>2</td>
<td>Al-graphite</td>
<td>Al-11wt.%Si-2.5wt.%Ni</td>
</tr>
<tr>
<td>3</td>
<td>Al-graphite</td>
<td>Al-12wt.%Si-1.5wt.%Cu-1.5wt.%Ni-1.5wt.%Mg</td>
</tr>
<tr>
<td>4</td>
<td>Al-graphite</td>
<td>Al-12wt.%Si-5.6wt.%Ni-3wt.%Cu</td>
</tr>
<tr>
<td>5</td>
<td>Al-graphite</td>
<td>Al-8wt.%Si-5.6wt.%Ni-3wt.%Cu</td>
</tr>
<tr>
<td>6</td>
<td>Al-glass</td>
<td>Al-3wt.%Mg.</td>
</tr>
<tr>
<td>7</td>
<td>Al-fly ash</td>
<td>Al-3wt.%Mg.</td>
</tr>
<tr>
<td>8</td>
<td>Al-shell char</td>
<td>Al-11.8wt.%Si-3wt.%Mg</td>
</tr>
<tr>
<td>Symbol</td>
<td>Composite system</td>
<td>Matrix composition</td>
</tr>
<tr>
<td>--------</td>
<td>------------------</td>
<td>--------------------</td>
</tr>
<tr>
<td>1</td>
<td>Al-graphite</td>
<td>Al-11.8 wt.%Si-1.5 wt.%Cu</td>
</tr>
<tr>
<td>2</td>
<td>Al-graphite</td>
<td>Al-11 wt.%Si-2.5 wt.%Ni</td>
</tr>
<tr>
<td>3</td>
<td>Al-graphite</td>
<td>Al-12 wt.%Si-1.5 wt.%Cu-1.5 wt.%Ni-1.5 wt.%Mg.</td>
</tr>
<tr>
<td>4</td>
<td>Al-graphite</td>
<td>Al-12 wt.%Si-5.6 wt.%Ni-3 wt.%Cu</td>
</tr>
<tr>
<td>5</td>
<td>Al-graphite</td>
<td>Al-8 wt.%Si-5.6 wt.%Ni-3 wt.%Cu</td>
</tr>
<tr>
<td>6</td>
<td>Al-glass</td>
<td>Al-3 wt.%Mg.</td>
</tr>
<tr>
<td>7</td>
<td>Al-fly ash</td>
<td>Al-3 wt.%Mg.</td>
</tr>
<tr>
<td>8</td>
<td>Al-shell char</td>
<td>Al-11.8 wt.%Si-3 wt.%Mg</td>
</tr>
</tbody>
</table>