A FORMULATION TO EVALUATE CAPACITY AND DELAY OF MULTILANE ROUNDABOUTS IN THE UNITED STATES FOR IMPLEMENTATION INTO A TRAVEL FORECASTING MODEL

by

Erik Lawrence Seiberlich

A Thesis Submitted in Partial Fulfillment of the Requirements for the Degree of Master of Science In Engineering at The University of Wisconsin-Milwaukee

December 2001
A FORMULATION TO EVALUATE CAPACITY AND DELAY OF MULTILANE ROUNDABOUTS IN THE UNITED STATES FOR IMPLEMENTATION INTO A TRAVEL FORECASTING MODEL

by

Erik Lawrence Seiberlich

A Thesis Submitted in Partial Fulfillment of the Requirements for the Degree of Master of Science In Engineering at The University of Wisconsin-Milwaukee December 2001
Modern roundabouts, first installed in England in the early 1960s, are becoming popular substitutes for signalized and stop controlled intersections in the United States. These facilities were originally introduced in order to solve the problems of the existing rotaries and traffic circles. Using the principle that entering traffic yields to circulating traffic, or the “give way” rule, roundabouts proved to be a much more efficient intersection than the rotaries, and in many cases, signalized intersections. This thesis presents a formulation for evaluating the capacity and delay of multilane roundabouts. The formulation was designed to be implemented into a travel forecasting model for application in the United States. Based on the study of many literature sources, this formulation uses the gap acceptance theory and evaluates entry lanes on a lane-by-lane basis. The utilization of the entry lanes is dependent on the equilibrium of delay, representing a user-optimal paradigm. Included in this thesis is a review of relevant and related literature, a detailed explanation of the formulation, and a
step-by-step example using realistic traffic data. The document also contains testing of the formulation and comparisons to a popular roundabout modeling software package.
I would like to thank my advisor, Dr. Alan Horowitz, for leading me through this thesis. His guidance was crucial for the production of this document. I would also like to thank Dr. Ed Beimborn and Dr. Zhong-Ren Peng for their valuable involvement and input.

I would like to recognize the support and encouragement provided by my family, mom and dad, Sues, Tom and Kevin. Special thank you to my sister Kristine for the advice and first hand experience on how to handle the whole thesis process. Finally, I would like to acknowledge someone who made much of my schooling possible. Thank you Grandma Hope for your assistance, in all respects.
Table of Contents

1 Introduction .................................................................................................. 5
  1.1 Basic Concepts of Roundabouts ................................................................. 5
  1.2 Basic Definitions ........................................................................................... 9
  1.3 Purpose/ problem ...................................................................................... 10
  1.4 Goals and Objectives ................................................................................. 11
  1.5 Hypotheses .................................................................................................. 12
  1.6 Approach ..................................................................................................... 15

2 Roundabout Capacity & Delay Models – A Literature Review ........ 17
  2.1.1 Roundabout History and Definition ....................................................... 18
  2.2 Analytical (Gap Acceptance) v. Empirical Regression (Geometric) ... 19
  2.3 Level of Analysis of Capacity and Delay ................................................ 24
    2.3.1 Equilibrium Traffic Assignment .............................................................. 27
  2.4 Parameters and Other Considerations .................................................... 28
    2.4.1 Driver Experience ...................................................................................... 29
    2.4.2 Circulating Roadway Lane Utilization ................................................... 30
    2.4.3 Critical Gap and Follow Up Time ........................................................... 31
  2.5 Roundabouts as an Element of Travel Forecasting Models ............. 34
  2.6 Summary ......................................................................................................... 36

3 Formulation Development .............................................................................. 37
  3.1 Network Topology ......................................................................................... 37
    3.1.1 Slip Lanes .................................................................................................... 40
  3.2 Components of the Formulation .................................................................. 41
  3.3 The Capacity Equation .................................................................................. 43
    3.3.1 Determination of Circulating and Entering Flows ................................ 44
    3.3.2 Determining Capacity of Each Entry Lane ............................................. 49
  3.4 The Delay Equation ....................................................................................... 52
    3.4.1 The Yield-Line Delay Equation ................................................................ 52
    3.4.2 Equilibrating Entry Lane Delay for Each Approach ............................. 55
  3.5 Formulation Summary .................................................................................. 57
  3.6 Formulation Example .................................................................................... 58
    3.6.1 Values for Variables and Parameters ...................................................... 58
    3.6.2 Step-by-Step Example of the Formulation ............................................. 59

4 Data Collected/ Results/ Model Testing ............................................................. 65
  4.1 The Traversal Delay Equation ...................................................................... 66
  4.2 Data Collection ............................................................................................... 70
  4.3 Sensitivity Testing .......................................................................................... 70
  4.4 Formulation Comparison .............................................................................. 78
  4.5 Summary ......................................................................................................... 82

5 Findings and Future Work ............................................................................ 83
5.1 Findings................................................................................................................................. 83
  5.1.1 Basis of the formulation ......................................................................................... 84
  5.1.2 Level of Analysis of Approach Lanes ................................................................. 85
  5.1.3 Quality and Integrity of the Formulation............................................................. 86
5.2 Future Work......................................................................................................................... 87
  5.2.1 Circulating Split ...................................................................................................... 87
  5.2.2 Driver Familiarity Factor ...................................................................................... 88
  5.2.3 Calibration and Comparison................................................................................. 89
6 Bibliography......................................................................................................................... 90
7 Appendix.............................................................................................................................. 93
## Table of Figures

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2-1</td>
<td>Circulating Traffic Tendency</td>
<td>30</td>
</tr>
<tr>
<td>2-2</td>
<td>One-way Street Grid</td>
<td>34</td>
</tr>
<tr>
<td>2-3</td>
<td>Series of T-intersections</td>
<td>35</td>
</tr>
<tr>
<td>3-1</td>
<td>Possible Network Topologies</td>
<td>38</td>
</tr>
<tr>
<td>3-2</td>
<td>Five-legged Roundabout</td>
<td>39</td>
</tr>
<tr>
<td>3-3</td>
<td>Selected Network Topology</td>
<td>40</td>
</tr>
<tr>
<td>3-4</td>
<td>Slip Lane Topology</td>
<td>41</td>
</tr>
<tr>
<td>3-5</td>
<td>Circulating Volume at Entry</td>
<td>47</td>
</tr>
<tr>
<td>3-6</td>
<td>Example Turning Movements</td>
<td>58</td>
</tr>
<tr>
<td>3-7</td>
<td>Adjusted Example Turning Movements</td>
<td>60</td>
</tr>
<tr>
<td>4-1</td>
<td>Circulating Volume Determination</td>
<td>68</td>
</tr>
<tr>
<td>4-2</td>
<td>Yankee Doodle and Lexington Turning Movements</td>
<td>71</td>
</tr>
<tr>
<td>4-3</td>
<td>Lexington and Yankee Doodle Results</td>
<td>72</td>
</tr>
<tr>
<td>4-4</td>
<td>Lexington and Yankee Doodle Flows</td>
<td>72</td>
</tr>
<tr>
<td>4-5</td>
<td>Lexington and Yankee Doodle Delays</td>
<td>73</td>
</tr>
<tr>
<td>4-6</td>
<td>Delay as a Function of Entering Volume</td>
<td>74</td>
</tr>
<tr>
<td>4-7</td>
<td>Delay as a Function of Circulating Volume</td>
<td>75</td>
</tr>
<tr>
<td>4-8</td>
<td>Delay as a Function of All Volume</td>
<td>76</td>
</tr>
<tr>
<td>4-9</td>
<td>Hypothetical Examples of Errors in the Formulation</td>
<td>77</td>
</tr>
<tr>
<td>4-10</td>
<td>Traversal Delay as a function of Circulatory Speed and Distance</td>
<td>78</td>
</tr>
<tr>
<td>4-11</td>
<td>Single Lane Roundabout Comparison</td>
<td>80</td>
</tr>
<tr>
<td>4-12</td>
<td>Two-lane Roundabout Comparison</td>
<td>81</td>
</tr>
</tbody>
</table>
Table of Equations

| Equation 3-1: Akcelik Base Capacity Model | 43 |
| Equation 3-2: PCE Adjustment | 45 |
| Equation 3-3: Arrival Flow Rate for Right Lane | 45 |
| Equation 3-4: Arrival Flow Rate for Left Lane | 46 |
| Equation 3-5: Outside Lane Circulating Flow at Entry | 47 |
| Equation 3-6: Inside Lane Circulating Flow at Entry | 47 |
| Equation 3-7: Adjusted Base Capacity Equation | 49 |
| Equation 3-8: Proportion of Unbunched Vehicles in the Circulating Stream | 51 |
| Equation 3-9: Base Delay Equation (Akcelik/Troutbeck) | 52 |
| Equation 3-10: Minimum Yield-Line Delay | 53 |
| Equation 3-11: Degree of Saturation | 53 |
| Equation 3-12: Performance Parameter for Average Overflow Queue | 53 |
| Equation 3-13: Adjusted Performance Parameter Equation | 54 |
| Equation 3-14: Overflow Parameter | 54 |
| Equation 3-15: Adjusted Yield-Line Delay | 55 |
| Equation 3-16: Basic Equilibrium | 56 |
| Equation 4-1: Base Traversal Delay | 66 |
| Equation 4-2: Free Flow Travel Time for a Leg | 67 |
| Equation 4-3: Total Traversal Delay | 68 |
| Equation 4-4: Average Traversal Delay | 69 |
| Equation 4-5: Average Facility and Queuing Delay | 69 |
1 Introduction

Modern Roundabouts were first introduced in the early 1960s in England. These facilities were introduced in order to solve the problems of the existing rotaries and traffic circles. Using the principal that entering traffic yields to circulating traffic, or the “give way” rule, roundabouts proved to be a much more efficient intersection than the rotaries, and in many cases, signalized intersections.

1.1 Basic Concepts of Roundabouts

Due to the technical nature of this thesis, it is necessary that basic concepts dealing with roundabouts be defined.

Roundabouts are intersections of two or more roads that are made up of a one-way circulating roadway that has priority over approaching traffic. The approaching traffic is controlled by yield signs, and can only make a right turn onto the circulating roadway. The only decision the entering motorist needs to make once they reach the yield line is whether or not a gap in circulating traffic is large enough for them to enter. The vehicles then exit the circulating roadway by making a right turn toward their destination.

Roundabouts are often confused with traffic circles or rotaries. These other facilities are often stop or signal controlled, and give priority to traffic entering
the intersection. Roundabouts have three main characteristics that identify them when compared to traffic circles:

- Offside priority or yield-at-entry – Roundabouts give vehicles in the circulating roadway the right of way. This is different than other uncontrolled, yield controlled or multi-way-stop controlled intersections that give priority to vehicles approaching from the right (North America). In addition to giving priority to vehicles already in the facility, roundabouts control the entering vehicles with a yield sign, not stop signs or traffic signals.

- Approach flare – Most roundabout approaches flare out at the entries and allow more vehicles to enter the circulating roadway at a more obtuse angle. This improves capacity, and allows entering vehicles to enter at similar speeds as the circulating vehicles unless a cue has developed at the entry. The size and angle of the flare is generally controlled by a raised splitter island that separates the entering and exiting traffic at an approach. This island also gives pedestrians a safe location to cross the approach in two stages.

- Deflection – This characteristic is the geometry of the facility that requires vehicles to slow down as they maneuver through the roundabout. The size
of the center island and angle of approach determine the deflection and potential speeds of entering and circulating vehicles.

In addition to these three primary requirements, there are other attributes common to roundabouts. Unlike traffic circles, where there can be pedestrian access to the center island, crosswalks for roundabouts are located on the approach legs, behind the yield lines. Also, parking is not allowed within the circulating roadway or at the entries of the approach legs, and all entering traffic is required to proceed around the roundabout in a counter-clockwise direction. (In some neighborhood traffic circles, a left turning vehicle is allowed to shorten its trip in front and to the left of the center island.)

The effect of the roundabout is that traffic is required to slow down to negotiate the curve around the center island, but unlike stop and signal controlled intersections, vehicles entering roundabout are not required to stop. This makes the facility more efficient under a broad range of traffic volumes, as motorists need only to find an acceptable gap for entrance.

In most cases roundabouts have been found to be much safer than other intersections. There are several reasons for this. First, roundabout design requires that vehicles slow to speeds of 15-30 miles per hour in order to maneuver the facility. Second, since all entering vehicles turn right, there is a
great reduction in the number of points of conflict compared to a conventional intersection. The reduction from 32 to 8 points of conflict lessens the chance for crashes, and when combined with the reduced speed, crash severity is further reduced. One element that may decrease safety is driver unfamiliarity with roundabouts. This factor would be most apparent with new roundabouts and will obviously diminish with time and driver experience.

The purpose of this thesis is to develop a capacity and delay formulation for roundabouts that can be used in a travel forecasting model. A travel forecasting model is a macroscopic program that uses origins and destinations and minimum disutility to determine vehicle paths. Macroscopic programs are more general and often deal with data at the traffic analysis zone level, where as microscopic models deal with individual vehicles. Microscopic models are very detailed and require that much more data and other information is provided for each facility, compared to macroscopic models. Microscopic models are used to analyze small areas, from one intersection to several corridors. The macroscopic models, for which this formulation is being developed, can deal with up to 20,000 nodes and 30,000 links.

With applications of this magnitude, it is important to keep the required data entry and design for each facility to a minimum. Also, the delay and capacity calculations for an individual intersection must be able to be completed in much
less than a second, for if the multilane roundabout is part of a very large model, it can not seriously slow down the entire application. For these reasons, one task of this activity will be to utilize only the absolutely required data and parameters, and not deal with less important variables or factors.

1.2 Basic Definitions

Several concepts and characteristics are unique to roundabouts. The following illustrates how certain characteristics of roundabouts will be referred to in this thesis.

Roundabouts are made up of a circulatory roadway and three or more approaches with entry and/or exit lanes. When looking at one approach, the vehicles entering the facility are described as the ‘entering flow at entry’ and are described by which lane they occupy, either the right or left lane. The vehicles on the circulatory roadway, passing in front of the approach are described as the ‘circulating flow at entry’ and are described as occupying the inside or outside lane, the inside lane being the closest to the center island.

Another term unique to roundabouts is the ‘slip lane’. A slip lane is a lane that allows right turning vehicles to do so without entering the circulatory roadway.
Several other terms must be established prior to getting any farther into the thesis. For the purpose of understanding, a hierarchy has been developed to explain the level at which an equation, or series of equations, relates to a model. A “model” will pertain to a simulation that evaluates many characteristics and performances of an intersection, corridor or network. Next is a “formulation”, which relates to a series of equations and variables that can evaluate certain performance characteristics. The smallest element is an “equation”. Several equations are required to make up a formulation.

1.3 **Purpose/problem**

The purpose of this thesis is to develop a formulation that can be implemented into a travel demand forecasting model. This formulation will evaluate the capacity and delay of a multilane roundabout. The primary problem that has been presented is to determine which of the myriad of prior formulations is most appropriate.

Different types of formulations have been developed. Some formulations evaluate the roundabout based on all the entry lanes of an approach, some treat each lane group as its own entity, and some use a lane-by-lane analysis based on other factors. Certain models are based on gap acceptance principals, while others are based on geometry and empirical regression formulas.
A gap acceptance formulation determines capacity of a roundabout using a minimum critical gap and follow up time that a driver would require when attempting to enter the circulating stream of traffic.

A regression model determines capacity based on certain geometrical characteristics of the facility and the circulating flow. This type of model is calibrated using empirical data from roundabouts at capacity conditions.

The task for this thesis will be to determine what type of formulation is most appropriate for evaluating roundabouts in the United States for the purpose of implementation into a travel forecasting model. Existing roundabout formulations were largely developed on other continents, and are based on the travel habits, vehicle characteristics and accepted driving practices in other countries, mainly in Western Europe and Australia.

1.4 Goals and Objectives

In order to fulfill the purpose of this thesis, several goals and objectives have been developed as a guide. These goals and objectives include the following:

- Based upon the review of published articles, design guides, and existing roundabout simulations, determine the appropriate basis for the desired
formulation; whether it should be a gap acceptance-based formulation, an empirical regression-based geometrical formulation, or possibly a combination of the two.

♦ Determine the level of complexity at which the capacity and delay will be evaluated. Determine whether the formulation will look at the roundabout as a single node, each approach as an individual node, or in more detail looking at queue length, travel time optimization, or lane groups which might affect the approach lane distribution.

♦ Identify any parameters that affect roundabouts in the United States differently than those overseas.

♦ Set values for new parameters that are developed for my formulation.

♦ Determine the appropriate level of detail and simplicity so that my formulation can be easily entered into a large-scale, macroscopic travel demand forecasting model.

1.5 Hypotheses

Many questions need to be answered before this model can be implemented into a travel forecasting model. These questions will act as hypotheses guiding the
evolution of the formulation so that it properly evaluates capacity and delay for roundabouts using driver and vehicle characteristics in the United States.

The questions are grouped into three categories. Each category further refines the development of the formulation. The first group helps define the basis for the formulation.

♦ Is a gap acceptance based formulation appropriate for use in a large-scale travel forecasting model? This type of formulation represents the behavior of drivers waiting for a gap large enough to enter into the circulating roadway.

♦ Is a regression formulation appropriate for use in a large-scale travel forecasting model? This type of formulation is primarily based on several geometric characteristics of the roundabout in addition to the circulating volume.

The next group of questions deals with the level of aggregation or complexity of the formulation.

♦ Is it most appropriate to analyze the roundabout by all approach lanes together? This method would aggregate all the movements from each approach, and give an overall capacity and delay for the approach without regard to different lanes.
Is it most appropriate to analyze the roundabout by lane groups? This method would treat each lane group of the approach individually, and provide different capacities and delays based on the entry and circulating flows for each group.

Is it most appropriate to analyze the roundabout on a lane-by-lane level? This type of analysis measures capacity and delay for each individual entering lane at each approach. Analyzing at this level would require specific information on the utilization of different lanes.

The next group of questions investigates the quality and integrity of the formulation, regarding the desired application.

Is there sufficient and relevant information for this formulation to be implemented into a U.S. based application? This question relates to whether or not the formulation appropriately relates to driver and vehicle characteristics found in the U.S.

Is there sufficient information so that this formulation can be implemented within a macroscopic travel forecasting model? Will this formulation function appropriately so that it correctly represents the characteristics of a real roundabout facility over a broad range of traffic conditions?

Are there any compromises that will defeat the validity of the delay estimates? Since the formulation is designed for a large network
macrosimulation, it must require much less data than those formulations in an intersection-specific simulation.

Answers for these questions will provide the framework and quality control for the capacity and delay formulation. The conclusion portion of the thesis (chapter 5) will answer these questions, and provide areas for which further study is needed for this topic.

1.6 Approach

This thesis consists of five chapters. The contents of chapters two through five are as follows.

Chapter 2 is the literature review. Chapter 2 discusses the findings of other research, and at the same time evaluates the relevance and appropriateness of the material for the purpose of this thesis.

Chapter 3 presents the development of the formulation. It proceeds through the steps, and details the different elements of this formulation. This chapter identifies the source of each element of the formulation and works through the capacity and delay evaluation process.
Chapter 4 discusses the gathering of data sets, and provides examples and results of the working formulation. In addition, this chapter provides the sensitivity testing of the formulation, and uses other tests to determine the fitness of this formulation for the purpose of implementation into a travel forecasting model. Finally, chapter 4 will compare results of this formulation with the formulation used in aaSIDRA, a popular roundabout simulation tool.

Chapter 5 presents conclusions of this research and provides suggestions for additional investigation on the topic.
2 Roundabout Capacity & Delay Models – A Literature Review

Since modern roundabouts were first introduced in the early 1960s, many different types of formulations have been developed in order to determine roundabout capacity and delay. This chapter addresses most of the different approaches taken to determine roundabout performance. The literature review will look at the different theories upon which these formulations are based, and the various equations that use a myriad of variables and parameters to estimate capacity and delay. These models come from many countries, but primarily from Australia and throughout Western Europe.

Many of the formulations constitute elements of different software packages that evaluate roundabouts and traffic corridors or networks on macroscopic or microscopic levels. With the information discussed in this literature review, it will be possible to determine which characteristics of roundabout performance estimation should be used for developing a formulation for determining the capacity and delay of American roundabouts for implementation into traffic forecasting models.

It has been determined that for the purpose of this thesis, a roundabout is a completely different entity than those facilities that people call traffic circles or rotaries. A roundabout has several characteristics unique from these other
circular intersections, and the formulations to be developed will only refer to roundabouts.

2.1.1 Roundabout History and Definition

The first circular intersection in the U.S. was Columbus Circle in New York City, built in 1905. Rotaries were built in the U.S. from the 1920s to the 1950s. In 1966, the mandatory “give-way” rule was adopted in England, requiring entering vehicles to yield to vehicles in the circulating roadway. In the 1970s and 80s roundabouts were first used in other European nations and Australia, and in 1990, the first roundabout was installed in the United States. (Ray and Rodegerdts, 2001).

There are several characteristics used to identify roundabouts distinctly from traffic circles and rotaries. Wallwork (1997) describes traffic circles as having square entries, and a stop control of the entry. Flannery, et al (1998), Myers (1994), and Oursten and Bared (1995) state the three main characteristics of roundabouts to be: yield on entry, deflection, and flared entries. In addition to these characteristics, defined previously in section 1, Ray and Rodegerdts (2001) also identify other elements that distinguish roundabouts from other circulatory roadway facilities. These elements include pedestrian access and crossing location, parking availability, and the direction of circulation on the roadway.
In roundabouts, pedestrians are only allowed to cross approaches. The crossing can be done across one direction of traffic at a time, with the splitter island providing a safe location for a pedestrian to wait in the middle of the street. Access is not allowed to the center island. Parking is not permitted in the circulatory roadway of roundabouts. Some other types of circulatory intersections allow parking. Traffic in the circulating roadway of American roundabouts always moves in the counter-clockwise direction. In some smaller traffic circles, traffic is allowed to pass in front and to the left of the center island instead of the 270-degree maneuver done in a roundabout.

2.2 Analytical (Gap Acceptance) v. Empirical Regression (Geometric)

There exist two distinct theories upon which roundabout capacity/ delay equations are based. These theories are the analytical or gap acceptance based method, and the empirical method, which is based on geometrics and regression.

In Kimber’s initial laboratory report (1980) he states that the dependence of entry capacity on circulating flow depends on the roundabout geometry. Kimber defines five geometric parameters which have an effect on the capacity. These are entry width and flare, the inscribed circle diameter (a line that bisects the
center island and the circulating lane twice), and the angle and radius of the entry.

In Kimber’s 1989 paper he states that gap acceptance is not a good estimator of capacity in the United Kingdom. He also continues that single-lane entries are the basis for the simplest case for gap acceptance models, while empirical models apply also to multilane entries. Kimber reasons that gap acceptance models do not increase capacity correctly when additional entry lanes are added.

♦ Perhaps Kimber’s reasoning in this publication was due to its creation date. Many new ideas have been put forth for how additional lanes affect capacity in a gap acceptance model.

Kimber makes two interesting comments in his paper, the first being that many circumstances exist where driver response to yield signs conforms to gap acceptance assumptions. However, he questions whether or not gap acceptance is a sufficient description of this interaction. The main flaw of the gap acceptance theory is that it poorly evaluates capacity for at-capacity roundabouts. Flannery et al (1998) comment that congested roundabouts are very scarce in the United States.

♦ Therefore, the empirical regression model might be difficult to use since it requires a saturated facility to be calibrated.
The second comment by Kimber is that because of driver behavior and geometric variation it is not safe to transfer theories from one country to another. Fisk, in a 1991 article, agreed that regression models should not be transferred from region to region, or between roundabouts of different geometrical configurations. Fisk writes that because a regression model requires a great deal of data for calibration, it may work well at a specific facility, but cannot be universal for many. Further, Fisk feels that gap acceptance models demonstrated reliable predictions for both capacity and delay of New Zealand roundabouts. Fisk believed that by changing vehicle class parameters or providing a range of critical gap values, gap acceptance modeling could be used in other locations.

Akcelik (ARR 321, 1998) contends that while Kimber objects to the “simple gap acceptance method”, the model presented for use in the SIDRA software package goes beyond the simple approach. One main addition to Akcelik’s gap acceptance approach is the modeling of the roundabout based on approach lane use. Furthermore, Akcelik writes that the method presented in his report improves capacity prediction during heavy flow conditions and especially for multilane roundabouts with uneven approach demands.

Many of the additional elements used in SIDRA are parameters used to enhance its basic gap acceptance theory. The parameters that deal with the entering traffic stream include the inscribed diameter, average entry lane width, the
number of circulating and entry lanes, the entry capacity (based on the circulating flow rate), and the ratio of the entry flow to the circulating flow. These additional model elements demonstrate the detailed nature of the SIDRA model. Another important component of Akcelik’s formulation is the identification of the dominant and subdominant entry lanes based on their flows. The dominant lane has the highest flow rate, and all others are subdominant. The purpose of this component is that dominant and subdominant entry lanes can have different critical gap and follow up times.

- The distinction between dominant and subdominant lanes appears to be quite important because vehicles using the leftmost entry lane must find a gap in both circulating lanes, as opposed to the right entry lane, which must only deal with traffic in the outer most circulating lane.

SIDRA also includes a passenger car equivalent (pce) for heavy vehicles. Akcelik (1997) recommended that pce per hour be used in place of vehicles per hour when the proportion of heavy vehicles surpassed 5%. Many other authors concurred with Akcelik’s recommendation. In their Roundabout Design Workshop (2001), Ray and Rodegertds explain that heavy vehicles primarily affect roundabout capacity with their size, not their slower acceleration and speed. The U.S. DOT’s Roundabout Guide (2000) suggests typical pce conversion factors for adjusting entering and circulating volumes. These include a 1.5 factor for recreational vehicles and buses, and a 2.0 factor for tractor-trailers.
Todd (1979) explained that U.S. roundabout capacity would be lower than that of other countries because of larger vehicle sizes.

♦ While this might be true, it would have little impact in a large travel forecasting model. Although a travel forecasting model allows different facilities to have different heavy vehicle adjustments, the larger size of American vehicles would create a minimal difference in delay over a large network.

♦ For the purpose of this thesis, the gap acceptance theory appears to be the most appropriate basis for the capacity and delay formulation. Since this formulation is designed to be implemented into a traffic forecasting model, it seems as though its capacity and delay equations should be similar to those used for other U.S. unsignalized intersections. The Highway Capacity Manual (HCM) (TRB 209, 1998) uses simple gap acceptance for two-way stop controlled intersections and roundabouts.

♦ Also, a single multilane roundabout is just one element in a traffic forecasting model. Since the user input should be kept to a minimum, an empirical regression based formulation would be too labor intensive.

Further investigation into which theory is more appropriate shows that the gap acceptance model is felt to be more transferable from country to country and
location to location than is the empirical regression model. List et al (1994) investigated multilane roundabouts in New York State using gap acceptance based models. They commented that it is possible to transfer capacity equations from overseas, but that it is important to develop unique formulations for U.S. conditions by modifying those developed abroad.

♦ That is the exact idea for this thesis.

2.3 Level of Analysis of Capacity and Delay

Roundabout capacity and delay analysis can be performed at several levels of detail. Akcelik (1998) mentions three methods by which capacity measurement can occur. These include analysis by total approach flow, as used in ARCADY, the British empirical regression based simulation. There is analysis by lane groups, as used for signalized intersections in the United States’ HCM, and there is lane-by-lane analysis, which is the method used in SIDRA.

Akcelik uses the lane-by-lane method for the purpose of allowing improved geometric modeling of the intersection. He points out that recognition of unequal lane utilization is important, because it affects the capacity and performance of the facility. Akcelik feels the best way to account for unequal lane use is by using dominant and subdominant lanes.
♦ It appears that modeling a multilane roundabout by combining the flows from all movements, and using the same parameters for both lanes would be inappropriate. Moreover, getting an average intersection delay across all legs would not serve the best purpose for the use of this formulation in any type of traffic modeling software. With consideration to the two statements above, the proper approach would be a distinct treatment of each lane of traffic.

This prior remark also agrees with Fisk’s (1991) statement that when an entering vehicle has the choice of lanes, there should be a method for determining the flow distribution by lane.

Fisk states that lane utilization for entering lanes should be determined using travel time minimization or by equalizing queue lengths. It is also mentioned that the right lane will be served at a faster rate than the left lane, and because of this, travel time minimization would be a better predictor.

♦ Akcelik’s use of dominant and subdominant lanes attacks this problem from a different angle, but would greatly increase the complexity to my formulation.

Fisk and Akcelik both recommend using a different critical gap and follow up time for each lane.
This is appropriate for the formulation being presented, because the vehicles using the left lane must essentially find a gap in the outside circulating lane at the same time it attempts to enter the gap in the inside circulating lane. The question then surfaces as how to determine what vehicles use which lane.

Akcelik’s model, using the dominant and subdominant lanes, looks at the turning movement of each approach. Considering exclusive and shared lanes, a lane utilization ratio is determined by the degrees of saturation of the lanes. Lane group capacity is then calculated, and eventually flow rate for each lane is determined.

While this process appears to fit the needs of the aaSIDRA simulation, it is not so appropriate for my formulation. Requirements for my formulation include low user input, very quick calculating and processing, and a preservation of simplicity.

Morlok (1978) states that behavioral studies of motorists indicate that motorists will choose their route based on the minimum travel time. This assertion compliments Fisk’s statement of minimizing travel time. Minimizing travel time appears to be the most appropriate method to determine lane utilization for this formulation. Fisk describes the problem to be a mini-traffic assignment problem.

For this model to be implemented into a travel forecasting model, the mini-traffic assignment would not necessarily work. Depending on the network
topology (discussed in section 3.1), the travel forecasting model will likely see all traffic entering the circulating stream as taking right turns. Because of this fact, it would be extremely difficult to determine lane use based on turning movement. The method that fits best for my formulation is to have equal delay in each lane of the approach.

- Equilibrating delay (see section 3.4.2) in each lane will minimize total travel time, as Fisk recommended, and it can be done without determining dominant/ subdominant lanes, or by figuring proportions of each turning movement and lane group. This maintains the simplicity of the formulation, while still attending to the issue of different parameters for the entering lanes.

2.3.1 Equilibrium Traffic Assignment

A brief discussion of the central ideas of equilibrium based traffic assignment is warranted since this is the method that is most promising for determining lane utilization. Morlok (1978) likens the traffic assignment equilibrium process to the equilibrium analysis of supply and demand in economic markets. However, for traffic assignment many elements (facility supply, traffic demand, travel time, mode choice) are considered to determine the disutility on a network. For most travel forecasting models there will be one element of disutility, and that will be travel time, which is dependent on delay.
As Morlik explains, the equilibrium theory is based on the desire of the motorist to travel the route with the lowest disutility (in this case travel time). If one route has a lower disutility than the other(s), motorists will use that route. If the volume of motorists on a particular route increases the disutility beyond that of an alternative route, motorists will switch to the new route, until another route has lower disutility. Traffic is assigned on the two (or more) routes so that the disutility or travel time is equal, and that there is no other unused route with a lower disutility than the routes being utilized.

2.4 Parameters and Other Considerations

As Akcelik (2001) suggested, a satisfactory capacity and delay formulation should include modeling driver yield behavior and geometry. The driver yield behavior is accounted for in the gap acceptance-based capacity and delay equations. The geometry delay is accounted for by the design of the roundabout in the travel forecasting model. This section will discuss the additional parameters and postulations that will be included in the product of this thesis; the capacity and delay formulation.
2.4.1 Driver Experience

Adjusting the volume for heavy vehicles has previously been discussed in section 1.2. Another factor affecting capacity is driver experience. List et al (1994) contend that when British or Australian capacity models are used in the U.S, they tend to overestimate throughput. This is due, they claim, to driver inexperience. British and Australian motorists are much more used to driving in these facilities than are most American drivers. Myers (1994) asks in his publication, "can American drivers adapt to roundabouts?" He answers his question with yes.

Fortuijn (1997) noted that capacity could be affected by whether the motorist is informed of the upcoming intersection prior to entering or while approaching the roundabout. This statement, as well as the statements by List and Myers identifies the need for a correction to the capacity relating to driver behavior.

♦ For this reason it seems appropriate to introduce a “driver familiarity” factor into the capacity and delay formulation. This “driver familiarity” parameter would assume that as driver experience and comfort with negotiating the roundabout increases, so will capacity. Since roundabouts are a relatively new type of intersection in the United States, motorists’ comfort level would seem to increase quickly since it is starting from near zero. Therefore, the “driver familiarity” factor should be used for new facilities and in locations that are always used by different drivers (airports, tourist attractions).
2.4.2 Circulating Roadway Lane Utilization

The most valuable information on lane use in the circulatory roadway comes from List et al (1994). They state that no more than 5% of circulating vehicles occupy the inside lane alongside a vehicle in the outside lane. It goes on to affirm that that does not mean the inside lane is unused, just that most motorists use it as a passing lane. Ray and Rodegertds explained in their Roundabout Design Workshop (2001) that vehicles in the circulating roadway tend to line up back bumper to front bumper, from the outside lane, going to the center (Figure 2-1).

![Figure 2-1: Circulating Traffic Tendency](image)

Consideration of both these statements leads to the belief that lane utilization in the circulatory lane is varied based on traffic volume entering the roundabout. Additionally, taking into account the 5% in List’s research, and the tendency explained by Ray and Rodegertds, the inside lane utilization would likely fall into the 15% - 25% range. This element of the capacity and delay formulation should be in the form of a parameter. There would be a default value of the parameter, but it may be changed based on observation of a particular roundabout, regional tendencies, or other considerations.
2.4.3 Critical Gap and Follow Up Time

There are a variety of ideas of what is the most correct critical gap for vehicles entering a roundabout. Cassidy et al (1995) state that it is not possible to directly observe the mean critical gap. This report also states that there is no evidence that a single-valued gap acceptance function cannot be used to model driver behavior reliably at a stop sign.

- Considering that traffic at a stop sign can perform three maneuvers, right turn, left turn and through, different sized gaps would be far more likely than in a roundabout where all motorists are taking right turns into the circulating roadway. Therefore, by the idea that if a single critical gap can be used for three maneuvers, it would only be more accurate for one maneuver.

Tian et al (2000) consider the many variables that can effect critical gap and follow up time. They state that geometry, turning movements, vehicle type and approach grade were found to affect these parameters. The delay a motorist has already incurred was also found to be a major factor affecting a motorist’s gap acceptance tendencies.

- Several of these factors are not applicable to the needed formulation. As stated previously, since there is only one possible turning movement, a right
turn into traffic, there is no need to vary these parameters based on traffic maneuver. Vehicle type has been addressed in passenger car equivalents, and for use in a large travel forecasting model, the 0.1 second adjustment for heavy vehicles recommended by Tian et al would be insignificant. Grades are not a part of most travel forecasting models. In addition, it is generally not a common practice to install a roundabout at a location with considerable vertical variation. The Federal Highway Administration (FHWA) (2000) states that it is not desirable to locate roundabouts where grades are greater than four percent. Therefore, it is assumed that most roundabouts will not deal with grade as a factor. The assertion that the delay a vehicle has incurred affects critical gap and follow up time is noted, but for the sake of keeping the simplicity of the formulation, it should return to Cassidy’s argument and use one value.

Still in question are the appropriate critical gap and follow up time. Akcelik (1998) introduced flow based estimations that determine the critical gap and follow up times. Akcelik documented a critical gap range of 2.2 to 8.0 seconds and a follow up headway of 1.2 to 4.0 seconds. To determine the appropriate magnitude of these parameters the dominant and subdominant lanes approach was used (discussed in section 2.3).
The Transportation Research Board (HCM 1997) presents its critical gap range as 4.1 to 4.6 seconds, and the follow up time as 2.6 to 3.1 seconds. These values, however, are for only single lane roundabouts. List et al (1994) determined the average critical gap to be from 2.8 to 4.0 seconds and the follow up time to range from 1.8 to 3.7 seconds. These values were most representative of the right lane. As stated earlier, the right lane will have a smaller critical gap and follow up time than the left lane, as the vehicles in the left lane have to cross the outside circulating lane. All of these gaps are considerably smaller than the recommended critical gaps and follow up times for two-way stop controlled intersections. The Transportation Research Board lists these as 6.9 and 3.3 seconds for a right turn onto a four-lane road, which is analogous to the circulatory roadway of a multilane roundabout.

Roundabout gaps and follow up times are smaller due to two reasons. The first is the ability for some vehicles to enter the circulating roadway without coming to a complete stop. If there are no queued vehicles in the entry lane, the yield control allows vehicles to only slow to the speed at which they can safely negotiate the roundabout. The second reason is the flare of the roundabout.

Tian et al (2000) observed that as the turn angle gets smaller, or closer to a through movement, the maneuver is easier and the critical gap tends to decrease.
Because roundabouts are flared, a smaller turning radius is created than would occur when turning right at a ninety degree intersection.

2.5  Roundabouts as an Element of Travel Forecasting Models

There has been little prior work done on how multilane roundabouts should be implemented into travel forecasting models. It is fairly easy to model single lane roundabouts in such a simulation. According Todd (1978) a roundabout is essentially a series of one-way streets that are yield controlled. He presents a grid of alternating one-way streets, in which a driver yields to traffic from the right at every second intersection (Figure 2-2).

![Figure 2-2: One-way Street Grid](image)

Wallwork (1997) and the City of Lincoln, Nebraska (2000) describe the roundabout as a series of T-intersections, where the top of the ‘T’ represents the circulating roadway, and the legs of the ‘T’ are the approach lanes, which yield to
the circulating flow (Figure 2-3). Each entry and exit is tangential to the circulatory roadway allowing the traffic to maneuver without stopping or slowing to a very low speed.

![Diagram of circulatory roadway and T-intersections]

**Figure 2-3: Series of T-intersections**

- The way roundabouts have been modeled was simply by using four T-intersections connected by a one-way road. These were designed the same way as network topology model 2, in section 3.1. The ease of evaluating single lane roundabouts is that there are few complex equations required. The necessary parameters are only critical gap and follow up time. There is no determination that has to be made regarding the use of different lanes, or the circulatory roadway. The driver familiarity factor presented in this thesis however, might be of value to existing single lane roundabouts. This factor can account for the potential decrease in capacity due to the inexperience of drivers in the United States.
2.6 Summary

- Based on this literature review, the foundation for the formulation to be illustrated in section 3 will work best following the equations used by Akcelik as presented in ARR 321. A gap acceptance based formulation will be more compatible with travel forecasting models, and there will be more consistency with what the user inputs for other types of intersections.

- Some type of modified lane-by-lane analysis appears to be the most appropriate way to model the lane utilization of entering traffic. Following Fisk’s suggestions, the lane utilization should be based on equilibrium of the delay for both lanes, with the right entry lane serving more vehicles than the left.

Several other parameters were identified and will be discussed further in the formulation development section. In addition, more detail will be given to the network topology that should accompany this formulation when implemented in a travel forecasting model.
3 Formulation Development

The overwhelming majority of research articles begin their analysis using single lane roundabouts and modifying their equations for multilane roundabout application. This formulation begins with the more complex multilane roundabout analysis, which then can be simplified when necessary for single lane usage. The formulation presented in this chapter will address determining the entering and circulating flows, capacity, and delay for each lane of the entering approaches of a single-lane or two-lane roundabout.

Prior to presenting the formulation, the network topology must be explained. Since this formulation has been developed to be used in a travel forecasting model, the proper network topology design is also required.

3.1 Network Topology

The term ‘network topology’ refers to the way the roundabout facility appears visually when building the network in a simulation. The chosen network topology can affect the design of the capacity and delay formulation. There are three designs that have been considered for use with this formulation. One design is to model the roundabout as a node bisecting two links (1). Another design is to have each entry as a node, with the circulatory roadway being a connection of those nodes (2). The third design is to use the previous node and
link method, but to have a series of one-way links, each representing an entry and circulating lane (3).

![Figure 3-1: Possible Network Topologies](image)

Each topology has its benefits and limitations. Topology 1, for instance, is the only topology that can differentiate between turning movements for each approach. In topologies 2 and 3, the only option for entering traffic is to turn right at the intersection, as the circulating roadway would be identified in the software as one-way. The benefits of topologies 2 and 3 include the ability to model geometry of the roundabouts. Instead of figuring geometrical features into the capacity or delay formulations, the links can be designed to mimic the circulatory roadway geometry of the roundabout. By adjusting the link speed limit to the intended or observed speeds of the circulating traffic, the model can deal with the time spent in the roundabout without any additional equations within the formulation.

Topology 3 is a complex arrangement that would remove complexity from the formulation. This design, however, could not accurately imitate the traffic
maneuvers that take place, as traffic could not freely change lanes in both the circulating and entry lanes. Topology 2 is the chosen network topology, as it fits best with the gap acceptance model that has been selected for this formulation. The need for the user to accurately model the link length is a shortcoming of this topology, but this topology is also the easiest for representing roundabouts with different numbers of approaches. (Figure 3-2)

![Figure 3-2: Five-legged Roundabout](image)

Figure 3-3 is a detailed illustration of what the network topology should resemble. The approach legs are two-way, two-lane streets, and the circulatory roadway is made up of one-way, two-lane links. The nodes are yield-controlled intersections, controlling the traffic on the entry approaches. For the network to correctly represent the actions of the facility, the speed limit on the circulatory links shall be set to 15-25 miles per hour; at whatever speed the roundabout was designed.
3.1.1 Slip Lanes

An element of some roundabouts is the slip lane or filter lane. Todd (1979) identifies these as lanes provided to give unimpeded movement to vehicles taking the first exit. This roundabout element can be handled both within the formulation and the network topology. For the formulation, the traffic that would use the slip lane (right turns in a four-legged roundabout) is simply not included in the entering flow. For the design of the network topology, a lane is simply added prior to the intersection of the entering link and the circulating link (Figure 3-4). It is critical that this lane is a one way link, and that left turns are prohibited at either intersection. It would be modeled essentially the same way as a freeway to freeway connector ramp.
3.2 Components of the Formulation

The initial analysis began with comparing the various capacity equations presented in publications from different regions in the world. There are different approaches that have been used to determine roundabout capacity. The two principal methods that have been used are gap acceptance and empirical regression models, which are based on facility geometry (see section 1.2). Since the desired formulation is to be used in travel forecasting models, it would seem appropriate to base it on similar principles as other unsignalized intersections. The Highway Capacity Manual evaluates unsignalized intersections (two-way and four-way stops, and single lane roundabouts) using gap acceptance theory equations. In addition, the regression formulations have many characteristics that make this type of formulation undesirable for use in network applications, while gap acceptance formulations can be manipulated to equilibrate delay for entry lanes by minimizing travel time. This is similar to the way most travel forecasting models determine traffic assignment.

Figure 3-4: Slip Lane Topology
The three primary methods that have been previously used in roundabout capacity and delay equations are discussed in section 1.3. These include analysis by total approach flow, analysis by individual movement groups (exclusive or shared lanes), and analysis based on each entering lane of the approach. The method used by Kimber (1980) in the United Kingdom is by total approach flow. This method is not sensitive to the different critical gaps and follow up times drivers use for each entering lane.

The analysis method that seems to be the most appropriate for this thesis is lane-by-lane. According to Akcelik, (1998) unequal lane utilization is an important factor that affects the capacity and performance of roundabouts. It is easiest to display the unequal lane utilization when modeling in a lane-by-lane fashion. This formulation, developed for implementation into travel forecasting models, determines lane utilization by equilibrating delay for each of the approach lanes based on user-optimal traffic assignment. This method simulates that each motorist will choose the approach lanes which allows them to enter the roundabout with the least delay.

It is known that driver experience and comfort is a factor in the capacity of intersections, freeways, and other transportation facilities. Because of this fact, it is expected that there will be a greater delay for each approach when the roundabout is newer, or in locations with new and changing users. This issue is
controlled by the driver familiarity factor. The driver familiarity factor that is part of the equation for circulating flow affects capacity, reducing the capacity of the approach as drivers become more familiar with roundabout. This is done by multiplying the circulating volumes by factors which are larger in for new roundabouts, or those located in areas with high use by tourists or drivers unfamiliar with the facilities.

Since the product of this research is a roundabout formulation intended for use in a travel forecasting model, it is imperative that it conforms to the format of these programs. The equations can use only the traffic information that will be provided as an output of the traffic assignment. There will however, be room to provide parameters for roundabout specific information such as geometry, driver familiarity, parameters controlling the use of the circulatory roadway lanes, and critical gap and follow up time.

### 3.3 The Capacity Equation

Equation 3-1 is used for the capacity portion of the formulation (Akcelik 1994). Several additional equations will be required in order to create inputs to this equation.

\[
Q_c = \left( \frac{3600}{\beta} \right) \ast \left( (1 - \Delta_c \ast q_c) + (0.5 \ast \beta \ast \Phi_c \ast q_c) \right) \ast e^{-\lambda \ast (\alpha - \Delta_c)}
\]

Equation 3-1: Akcelik Base Capacity Model
Where,
\[ \beta = \text{Follow up headway (seconds/vehicle)} \]
\[ \alpha = \text{Critical gap (seconds/vehicle)} \]
\[ \Delta_c = \text{Intrabunch headway (seconds/vehicle)} \]
\[ q_c = \text{Circulating flow at entry (pce/hour)} \]
\[ \Phi_c = \text{Proportion of unbunched vehicles in the circulating stream} \]
\[ \lambda = \text{Parameter in the exponential arrival headway} \]
\[ Q_e = \text{Capacity of a single entry lane (pce/hour)} \]

The process first begins with determining the entering and circulating flows. Therefore, the flow element of the formulation should precede the capacity equation.

### 3.3.1 Determination of Circulating and Entering Flows

Much of the uniqueness of this formulation pertains to the flow equations. A travel forecasting model attempts to find the path of minimum disutility from origin to destination. Because this formulation is to be used at a macroscopic level, a user is not required to enter turning movements. For the purpose of detailing the flow analysis however, the formulation will be presented as a stand-alone facility.

<table>
<thead>
<tr>
<th>Passenger Car Equivalent Factors</th>
<th>Private Automobile</th>
<th>RV/Bus/Delivery Truck</th>
<th>Tractor-Trailer</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>1.5</td>
<td>2.0</td>
<td></td>
</tr>
</tbody>
</table>

*Table 3-1: Passenger Car Equivalents*

The first step of the formulation is to modify the initial turning movements to represent passenger car equivalents (pce). The factors used in this formulation are shown in Table 3-1.
Equation 3-2 accomplishes the adjustment for heavy vehicles.

\[
VOL' = VOL \cdot P_t \cdot 2 + VOL \cdot P_r \cdot 1.5 + VOL \cdot (100 - (P_t + P_r))
\]

Equation 3-2: PCE Adjustment

Where,
- \(VOL\) = Turning movement volume (vehicles/ hour)
- \(P_t\) = Proportion of vehicles that are tractor-trailers
- \(P_r\) = Proportion of vehicles that are RVs, buses, or delivery type trucks
- \(VOL'\) = Adjusted turning movement volume (passenger car equivalent/ hour)

The next step of the formulation is to calculate the approach flow for each lane. This is done by summing the adjusted turning movement volumes for each approach and multiplying the total approach flow by the split for each lane. A new variable is required in order to perform this split. The entering split \((P_{ed})\) is a variable that represents the proportion of the entering vehicles that are using the right side approach lane. This variable can change iteratively based on the equilibrium of entry lane delay, which will be explained in more detail later in this section. The arrival flow rates are calculated in Equation 3-3 and Equation 3-4.

\[
q_{ar} = \sum_d VOL'_e \cdot P_{ed}
\]

Equation 3-3: Arrival Flow Rate for Right Lane

\[
q_{al} = \sum_d VOL'_e \cdot (1 - P_{ed})
\]
**Equation 3-4: Arrival Flow Rate for Left Lane**

Where,

- $V_{OL}^{'e}$ = Adjusted approach volume (sum of four approach movements) (pce/ h)
- $P_{ed}$ = Entering split (proportion in right lane) for direction for approach ‘d’
- $q_{ar}$ = Arrival flow rate for the right lane (pce/ h)
- $q_{al}$ = Arrival flow rate for the left lane (pce/ h)
- $d$ = Direction of approach (NB, SB, EB...)

Once entering or arrival flow has been calculated, the next step is to calculate the circulating volumes at each approach. There are several stages to this; the first is to determine what flows from each other entry will pass by this entry on the circulatory roadway. Figure 3-5 shows the result of this determination. For instance, for the vehicles that will pass by the northbound approach include the southbound left (g), the eastbound left (j) and thru (k), and the U-turns from the westbound (2), southbound (3) and eastbound (4) approaches. Each movement that passes an approach is an adjusted circulating volume ($V_{OL}^{'}c$) for that approach.

This step is handled by the traffic assignment in a travel forecasting model, but is demonstrated for understanding the evaluation of a roundabout.
The circulating flow at entry for a given approach lane is calculated by Equation 3-5 and Equation 3-6.

\[ q_{cr} = \sum_d VOL'_c \cdot P_c \cdot \Psi \]

Equation 3-5: Outside Lane Circulating Flow at Entry

\[ q_{cl} = \sum_d VOL'_c \cdot \Psi \]

Equation 3-6: Inside Lane Circulating Flow at Entry

Where,
- \( VOL'_c \) = Adjusted circulating flow at a given approach (pce/h)
- \( P_c \) = Circulating split (proportion of vehicles in the outside lane)
- \( q_{cr} \) = Circulating flow at entry for the right lane (pce/h)
\( q_d \) = Circulating flow at entry for the left lane (pce/h)
\( \Psi \) = Factor based on driver familiarity
\( d \) = Direction of approach (NB, SB, EB...)

The driver familiarity parameter (\( \Psi \)) affects the total circulating volume, which ultimately affects the capacity of an entry lane. The value of this parameter should decrease as driver familiarity increases, eventually having no effect on the circulating traffic when drivers are comfortable. Recommended values range from about 1.20 for uncomfortable drivers to 1.00 once drivers become acclimated to the facility. This parameter is based on present day conditions, and will require adjustment as roundabouts become more common in the United States. Other considerations, such as a location near an airport or tourist attraction where motorists are more likely to be unfamiliar with the facility might also apply. The most appropriate method would be to calibrate this parameter based on field data.

The circulating (or outside) split variable \( (P_c) \) measures the proportion of circulating vehicles that occupy the outside lane. This parameter is a user-controllable parameter that can range from 0.5 to 1.00. The latter should be used for roundabouts with low capacities or with a single circulating lane. The lower limit of 0.5 assumes that at capacity the inside lane would be fully utilized. Unless there are extraordinary circumstances (a crash or stalled vehicle in the outside lane) this lane should not have more traffic than the outside lane. The
recommended default value is 0.8, representing that 80% of the traffic travels in the outside lane. This value is based on review of literature and previous studies, which can be found in section 1.4.2.

The steps and equations in this section have now produced the necessary variables for the capacity model.

3.3.2 Determining Capacity of Each Entry Lane

As stated previously, the gap acceptance capacity model (Equation 3-1), as presented in ARR 321 by Akcelik, will be used for this formulation. Equation 3-7 shows this equation with minor alterations so that $q_c$ is represented in the equation as passenger car equivalents per second. This adjustment is made by dividing the circulating flow by 3600, the number of seconds in an hour.

$$Q_e = \left(\frac{3600}{\beta}\right) \ast \left((1 - \frac{\Delta_c \ast q_c}{3600}) + \left(\frac{\beta \ast \Phi_c \ast q_c}{2 \ast 3600}\right)\right) \ast e^{-\frac{\lambda}{3600} \ast (\alpha - \Delta_c)}$$

Equation 3-7: Adjusted Base Capacity Equation

Where,
- $\beta$ = Follow up headway (seconds/ vehicle)
- $\alpha$ = Critical gap (seconds/ vehicle)
- $\Delta_c$ = Intrabunch headway (seconds/ vehicle)
- $q_c$ = Circulating flow at entry (pce/ hour)
- $\Phi_c$ = Proportion of unbunched vehicles in the circulating stream
- $\lambda$ = Parameter in the exponential arrival headway
$Q_e =$ Capacity of a single entry lane (pce/hour)

This equation is used to find the capacity of each entry lane of a given approach.
Different values for some of the parameters are used for the different entry lanes.
Critical gap and follow up headway will have lower values for the right entry lane than the left, as it will take a vehicle longer to get into the inside lane. The other alterable variable, the circulating flow at entry ($q_{c_l}$ and $q_{c_r}$), which was calculated in the preceding section, will differ based on the particular lane.

The recommended values for $\alpha$ and $\beta$ are 3.5 and 3 for the right entering lane, and 4.5 and 3.5 for the left entering lane. All of these values are in seconds per vehicle. These could be changed by the user if necessary, based on observed data or for situation specific cases, but the right entry lane shall always have lower values for these variables than the left entry.

Two elements of this equation, $\Phi_c$ and $\Delta_c$, are fixed parameters. $\Phi_c$ represents the proportion of unbunched vehicles in the circulating stream. Used as a calculated variable in Akcelik’s capacity equation, this will be a fixed parameter with the value of 0.55 for this formulation. This is done to maintain simplicity for users and to reduce processing time. In ARR 321, Akcelik provides an equation for $\Phi_c$ for two lanes of circulation (Equation 3-8). 0.55 is the resulting value when the circulating flow is near capacity.
\[ \Phi_c = e^{-3.0 \frac{q_e + q_{cl}}{3600}} \]

Equation 3-8: Proportion of Unbunched Vehicles in the Circulating Stream

\( \Delta_c \) represents intrabunch headway. Akcelik gives a value of 1.2 seconds for circulatory roadways with two lanes, but states that that value assumes equal flows. He gives further equations for cases of unequal flows. However, these equations will be left out of this formulation for the purpose of keeping it less intricate, and the value of 1.2 will be used.

The remaining element for this equation is \( \lambda \). This is a parameter in the exponential arrival headway, that according to Tanner (1962, 1967) can be assumed to be equal to the circulating flow. For the purposes of this formulation, \( \lambda \) will still be represented, and not displayed as \( q_c \). In addition, \( \lambda \) must be converted to pce/sec, by dividing it by 3600.

The result of Equation 3-7 is an entry lane capacity, given in passenger car equivalents per hour. For a four legged, two lane roundabout, there will be eight individual capacities. The next step in the process is to determine the average delay for a vehicle passing through the roundabout.
3.4 The Delay Equation

The delay portion of the formulation is made up from parts of the yield-line delay equation. This equation determines the average delay per vehicle of a given entry lane, based on queuing and gap acceptance into the circulating flow. The travel forecasting model automatically simulates the traversal delay, based on the size and design speed of the circulatory roadway.

3.4.1 The Yield-Line Delay Equation

The basis of the yield-line delay equation is the extended form of the Akcelic-Troutbeck equation, found in ARR 321. This is shown below as Equation 3-9.

\[
D = d_m + 900T_f \left( z + \left( z^2 + \frac{8 \times K_d \times \chi}{Q_e \times T_f} + \frac{16 \times K_d \times N_i}{(Q_e \times T_f)^2} \right)^{0.5} \right)
\]

Equation 3-9: Base Delay Equation (Akcelik/Troutbeck)

Where,
- \(d_m\) = minimum yield-line delay (seconds)
- \(T_f\) = duration of the analysis (hours)
- \(z\) = performance parameter for average overflow queue
- \(K_d\) = overflow parameter
- \(\chi\) = degree of saturation
- \(N_i\) = initial queued demand at start of the analysis period (pce)
- \(Q_e\) = capacity of a single entry lane (pce/ h)
- \(D\) = average yield line delay (seconds)
There are several additional equations that go into determining the variables and parameters for this equation. The minimum yield-line delay is dependent on the capacity of the entry lane, and is shown in Equation 3-10.

\[ d_m = \frac{3600}{Q_e} \]

**Equation 3-10: Minimum Yield-Line Delay**

The overflow term parameter \( z \), is dependent on the degree of saturation \( x \), as well as the initial queue, and capacity of the entry lane. In determining \( z \), \( x \) must first be determined. The degree of saturation is calculated using the arrival flow and the capacity, as shown in Equation 3-11.

\[ x = \frac{q_a}{Q_e} \]

**Equation 3-11: Degree of Saturation**

Where,
\( q_a = \) arrival flow rate for a given entry (pce/ h)

With the degree of saturation calculated, the performance parameter \( z \) can be determined. Equation 3-12 is used to calculate the performance parameter.

\[ z = x - 1 + \left( \frac{2 \ast N_i}{Q_e \ast T_f} \right) \]

**Equation 3-12: Performance Parameter for Average Overflow Queue**
It is important to recognize that because this formulation is for use in a travel forecasting model, the initial queued demand at the start of the analysis ($N_i$) will be 0 (zero). Therefore, the second term of equation will zero out, leaving Equation 3-13 for determining the performance parameter.

$$z = x - 1$$

**Equation 3-13: Adjusted Performance Parameter Equation**

The overflow parameter $K_d$, is a parameter in the yield-line delay equation, and will be fixed at 1.0 for this formulation. Equation 3-14 shows the rationale for $K_d = 1.0$.

$$K_d = \frac{d m * Q_e}{3600} = -\frac{3600 * Q_e}{3600} = 1.0$$

**Equation 3-14: Overflow Parameter**

The final issue for this equation deals with the capacity variable. $Q_e$ is determined with the capacity equation part of the formulation, as shown in section 3.2.2. For the yield-line delay equation to work properly, $Q_e$ will be limited to a minimum of 33 pce/ h. This limit of 33 pce/ h is taken from the 1985 HCM minimum capacity for stop and yield signs. It is also justified based on Kimber’s “priority reversal” concept (1989). Kimber states that at high saturation levels, where low speeds would be evident in the circulating roadway, entering vehicles would edge into traffic to create a gap. Therefore,

$$Q_e = Q_e \quad \text{if } Q_e \geq 33$$
\[ Q_e = 33 \quad \text{if} \quad Q_e < 33 \]

With these alterations and additional equations, it will be possible to calculate delay for each entering lane of each approach. Equation 3-15 is the adjusted delay equation to be used for this formulation.

\[
D = \frac{3600}{Q_e} + 900T_f \left( x - 1 + \left( \frac{8 \times K_d \times x}{Q_e \times T_f} \right)^5 \right)
\]

Equation 3-15: Adjusted Yield-Line Delay

The result of Equation 3-15 will be a yield-line delay for each entry. The method this thesis presents to determine lane utilization of the entry lanes at each approach is by equilibrating the delay. This method is shown in the following section.

3.4.2 Equilibrating Entry Lane Delay for Each Approach

Lane utilization for this formulation is based on delay equilibrium. The theory behind this basis is that motorists will choose their entry lane based on minimizing their travel time. If it seems to a motorist that the lane other than the one they are in is serving vehicles faster, they will tend to move to that lane. If that motorist is wrong, other vehicles will fill their former spot, and equilibrium will occur.
Basically this can be shown as,

\[ D_r (P_{ed}) = D_l (1 - P_{ed}) \]

Equation 3-16: Basic Equilibrium

Equilibrium of delay for both entry lanes of an approach is achieved by adjusting the entering split. The right entry lane will always have a higher capacity than the left entry lane. This is because critical gap and follow up time is lower for the vehicles in the right lane, and the motorists in the left lane must deal with more circulating traffic. Therefore, the proportion of entering vehicles using the right entry lane will range from 0.5 to 1.0.

By equilibrating delay for both entry lanes, each approach will have exactly one value for delay. Although this could make the formulation appear to not consider the different lane characteristics, that inference is false, as the lane utilization will show different entering flows for each lane.

The iterative process to achieve Equation 3-16 would take a great amount of time if it were performed by hand, but in a spreadsheet or travel forecasting model, it can and must be performed in much less than one second. In a travel forecasting model only yield-line delay must be determined, as the traversal delay is
simulated by the travel time in the facility using the topology discussed in section 3.1.

3.5 **Formulation Summary**

There are three main elements to the formulation presented in this thesis. Each element is summarized below.

**Flow Determination:**
- This part of the formulation includes the heavy vehicle adjustment, the driver familiarity adjustment for user experience, and the circulating and entering splits. The entering split is a key part of equilibrating delay to determine approach lane utilization.

**Capacity Equation:**
- The capacity equation element uses the circulating flow determined in the prior element to calculate the capacity of each entry lane. This is based on the critical gap and follow up time, as well as bunching parameters.

**Delay Equation:**
- If this formulation is implemented into a travel forecasting model the only necessary part of the delay equation is the yield-line delay. This is found using the entry lane capacity established in the prior element and the degree of saturation, which also incorporates entering flow. The yield-line delays for each lane of an approach are then equilibrated based on the proportion of entering vehicles using each lane. When using this formulation outside of a
travel forecasting model, traversal delay is computed to determine the average time a vehicle from a given approach spends negotiating the facility. This is presented for testing and comparison purposes in chapter 4. For total delay, the yield-line and traversal delays are summed for each approach.

3.6 Formulation Example

The following sections illustrate how the different equations in my formulation are used to calculate a delay. There is first a section showing the values used for the variables and parameters, followed by a section that walks through the formulation step by step.

3.6.1 Values for Variables and Parameters

<table>
<thead>
<tr>
<th></th>
<th>NB</th>
<th>SB</th>
<th>EB</th>
<th>WB</th>
</tr>
</thead>
<tbody>
<tr>
<td>Left</td>
<td>170</td>
<td>255</td>
<td>90</td>
<td>225</td>
</tr>
<tr>
<td>Through</td>
<td>300</td>
<td>375</td>
<td>420</td>
<td>390</td>
</tr>
<tr>
<td>Right</td>
<td>420</td>
<td>200</td>
<td>250</td>
<td>155</td>
</tr>
<tr>
<td>U-turn</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Figure 3-6: Example Turning Movements

Heavy Vehicles: semi trucks - 3%, RV/ bus - 2%

Driver Familiarity Factor ($\psi$): 1.075

Critical Gap ($\alpha$): right lane - 3.5 seconds/ veh, left lane - 4.5 seconds/ veh

Follow-up Time ($\beta$): right lane - 3.0 seconds/ veh, left lane - 3.5 seconds/ veh
Intrabunch Headway ($\Delta_c$): 1.2 seconds/veh

Proportion Unbunched in Circulatory Stream ($\Phi_c$): 0.55

Duration of the Analysis ($T_i$): 0.25 hours

Overflow Parameter ($K_d$): 1.0

### 3.6.2 Step-by-Step Example of the Formulation

This example will focus on the northbound approach. In order to find the entering and circulating flows however, all traffic movements will be used for the flow calculations. The first step is to adjust the flows for heavy vehicles.

$$VOL' = VOL \cdot P_t \cdot 2 + VOL \cdot P_r \cdot 1.5 + VOL \cdot (100 - (P_t + P_r))$$

The PCE adjustment equation with the volumes for northbound lefts, throughs and rights are:

$$177 = 170 \cdot 0.03 \cdot 2 + 170 \cdot 0.02 \cdot 1.5 + 170 \cdot (100 - (0.05))$$

$$312 = 300 \cdot 0.03 \cdot 2 + 300 \cdot 0.02 \cdot 1.5 + 300 \cdot (100 - (0.05))$$

$$437 = 420 \cdot 0.03 \cdot 2 + 420 \cdot 0.02 \cdot 1.5 + 420 \cdot (100 - (0.05))$$

The figure below shows the PCE adjusted turning movements.
The next step is to calculate the approach flow for each lane. This will be demonstrated for the northbound approach only.

\[ q_{ar} = \sum d \text{VOL}'_e \times P_{ed} \quad q_{al} = \sum d \text{VOL}'_e \times (1 - P_{ed}) \]

The right and left entry lane equations with the adjusted volumes substituted:

\[ q_{ar} = 926 \times P_{en} \quad q_{al} = 926 \times (1 - P_{en}) \]

The \( P_{en} \) variable is unknown until the equilibration step. For purposes of this example, it will be assumed that 95% of the traffic uses the right lane until the first equilibration iteration is done.

\[ q_{ar} = 926 \times 0.95 \quad q_{al} = 926 \times (1 - 0.95) \]

\[ q_{ar} = 880 \quad q_{al} = 46 \]
The next step is to determine the circulating flow at the northbound approach. Referring back to Figure 3-5, it has been determined that this flow is made up of the southbound lefts, and eastbound lefts and throughs, as well as all U-turns, with the exception of northbound. The equations for the outside and inside lane are shown with and without the example numbers.

\[ q_{cr} = \sum_d VOL'_c * P_c * \Psi \quad q_{cl} = \sum_d VOL'_c * \Psi \]

\[ q_{cr} = 796 * 0.8 * 1.075 \quad q_{cl} = 796 * 1.075 \]

\[ q_{cr} = 685 \quad q_{cl} = 856 \]

Now that values have been calculated for entering and circulating volumes, the capacity of each lane should be determined. The capacity equation is shown, beside the computed capacities for each lane.

\[ Q_e = \left( \frac{3600}{\beta} \right) * ((1 - \frac{\Delta_c * q_c}{3600}) + (\frac{\beta * \Phi_c * q_c}{2 * 3600})) * e^{\frac{\lambda}{3600} * (\alpha - \Delta_c)} \]

\[ Q_{er} = \left( \frac{3600}{3.0} \right) * ((1 - \frac{1.2 * 685}{3600}) + (\frac{3.0 * 0.55 * 685}{2 * 3600})) * e^{\frac{685}{3600} * (3.5 - 1.2)} \]

\[ Q_{el} = \left( \frac{3600}{3.5} \right) * ((1 - \frac{1.2 * 856}{3600}) + (\frac{3.5 * 0.55 * 856}{2 * 3600})) * e^{\frac{856}{3600} * (4.5 - 1.2)} \]
\[ Q_{er} = 719 \quad Q_{el} = 443 \]

Now that the capacities for each lane have been determined, the initial delays can be determined using the delay equation. The delay equation is shown, beside the computed delays for the right and left entry lanes at a 0.95 to 0.05 split, since the first iteration is done with 95% of entering traffic using the right lane.

\[
D = \frac{3600}{Q_e} + 900T_f (x - 1 + ((x - 1)^2 + \frac{8 * K_d * x}{Q_e * T_f})^5
\]

\[
D_r = \frac{3600}{719} + 225*(x - 1 + ((x - 1)^2 + \frac{8 * 1 * x}{719 *.25})^5
\]

\[
D_l = \frac{3600}{443} + 225*(x - 1 + ((x - 1)^2 + \frac{8 * 1 * x}{443 *.25})^5
\]

The degree of saturation \(x\) can be computed using the following equation.

\[
x = \frac{q_a}{Q_e} \quad x_r = \frac{880}{719} \quad x_r = 1.2239
\]

\[
x_l = \frac{46}{443} \quad x_l = 0.1038
\]
The following equations include the values for the degrees of saturation for each lane, followed by the delays for each lane.

\[ D_r = \frac{3600}{719} + 225 \times (1.2239 - 1 + ((1.2239 - 1)^2 + \frac{8 \times 1 \times 1.2239}{719 \times .25})^5 \]

\[ D_l = \frac{3600}{443} + 225 \times (0.1038 - 1 + ((0.1038 - 1)^2 + \frac{8 \times 1 \times 0.1038}{443 \times .25})^5 \]

\[ D_r = 128.15 \quad D_l = 9.07 \]

The equation used for equilibrating delay follows. This equation indicates that by changing the entering split, equal delays for both lanes can be achieved.

\[ D_r (P_{ed}) = D_l (1 - P_{ed}) \]

After several iterations, or if using a spreadsheet, it would be determined that the correct entering split is between 0.65 and 0.66 (0.6575604).

Using this entering split (0.6575604), the entering flows are computed.

\[ q_{ar} = 926 \times .65756 \quad q_{al} = 926 \times (1 - .65756) \]

\[ q_{ar} = 609 \quad q_{al} = 317 \]
With the new entering flows for each lane, new degrees of saturation can be computed. These equations are shown.

\[ x_r = \frac{609}{719} \quad x_r = 0.8470 \quad \frac{x_i}{443} = 0.317 \quad x_i = 0.7156 \]

Now the delays can be computed using these new entering flows for each lane.

\[ D_r = \frac{3600}{719} + 225 \times (0.8470 - 1 + ((0.8470 - 1)^2 + \frac{8 \times 1 \times 0.8470}{719 \times 0.25})^{0.5} \]

\[ D_l = \frac{3600}{443} + 225 \times (0.7156 - 1 + ((0.7156 - 1)^2 + \frac{8 \times 1 \times 0.7156}{443 \times 0.25})^{0.5} \]

\[ D_r = 26.20 \quad D_l = 26.06 \]

The spreadsheet used for this formulation equilibrates delay to 26.11 seconds per vehicle for this example. However, for the purposes of hand calculation, the right lane delay of 26.20 s/ v and left lane delay of 26.06 s/ v is as close as can be expected considering rounding and significant digits.
4 Data Collected/Results/Model Testing

In order to show the functionality of this two-lane roundabout capacity and delay formulation several tests have been performed. These tests use data from the intersections of two lane highways, as well as a comparison to data and results from two actual aaSIDRA applications.

As mentioned in chapter 3, a traversal delay equation will be used to determine the average time it would take a motorist to travel from entrance to his desired exit of the roundabout facility. Not part of the desired formulation, this component of the research will allow this formulation to be tested and compared to other existing formulations.

When implemented into a travel forecasting model the traversal delay will be simulated by determining the travel time based on the geometry of the circulating roadway and the design speed. This is done using a weighted average of the traversal delay for each leg of the circulatory roadway, using free travel time, volume-to-capacity ratio, and entering flow. The equation and methodology for the purpose of comparing the traversal delay is presented below.
4.1 The Traversal Delay Equation

The traversal delay can be defined as the time it takes a vehicle to negotiate the roundabout facility; that is, the time elapsed between entrance into the circulating lane and exit out of the roundabout. This term of the total average delay is not needed when implementing the formulation into a travel forecasting model. The model will automatically determine traversal delay, as it is part of the overall travel time determination. For the purpose of comparison and demonstration, however, this section of the thesis will explain this portion of the delay computation. Equation 4-1 illustrates the equation for traversal delay, adapted from the Bureau of Public Roads curve.

\[
 t_d = t_o \times (1 + 0.8 \times \left(\frac{v}{c}\right)^5 )
\]

Equation 4-1: Base Traversal Delay

Where,
\( t_o \) = free flow travel time for a given leg of the circulatory roadway (seconds)
\( v \) = volume of the circulatory roadway between entry and exit (v/h)
\( c \) = capacity of the circulatory roadway (1 lane = 1900, 2 lane = 2375) (v/h)
\( t \) = travel time for a given leg of the circulatory roadway (seconds)
\( d \) = the particular leg (NB-WB, WB-SB, SB-EB...)

The constant 0.8 and the exponential parameter 5.0 are adopted from the suggested parameters presented in NCHRP #365 (TRB, 1998). These are empirical constants. Free flow travel time of a leg of the circulating roadway is
determined by dividing distance by free speed for the circulating roadway. This is shown in Equation 4-2.

\[
 t_o = \frac{W_{cd}}{c_s \times 1.46667}
\]

Equation 4-2: Free Flow Travel Time for a Leg

Where,
\(W_{cd}\) = distance from a given entry flare to the next entry flare (feet)
\(c_s\) = free speed of the circulatory roadway (miles per hour)
1.46667 is a conversion factor from miles per hour to feet per second

Determining the volume for this equation is done in a similar way as shown in Figure 3-5 in section 3. The difference is that the circulating volume of a leg includes vehicles that exit at the next egress point, whereas the number of circulating vehicles at entry does not include vehicles that exit before the entry, and do not circulate past. Figure 4-1 shows the process for finding this variable.

For instance, the vehicles that use the northbound to eastbound leg include all of the northbound traffic (a, b, c), the eastbound thru (k) and left (j), the southbound left (g) and all of the U-turn movements. Each box in Figure 4-1 represents a different volume (\(v\)), for which there can be a different free flow travel time (\(t_o\)).
Once the travel times for each leg have been computed with equation 15, the next step is to find total delay for that approach. This is done by taking each traversal delay and multiplying it by the flow from the particular approach that uses that leg. Equation 4-3 illustrates this computation.

\[
\begin{align*}
    t_i &= t_1 \times (l, h, r, u) + t_2 \times (l, h, u) + t_3 \times (l, u) + t_4 \times (u)
\end{align*}
\]

Equation 4-3: Total Traversal Delay

Where,
\( t_1 = \) traversal delay of the first leg to the right of the entry (seconds)
\( t_2 = \) traversal delay of the second leg to the right of the entry (seconds)
where,

\[ D_t = \frac{t_t}{VOL_e} \]

Equation 4-4: Average Traversal Delay

The final step of the formulation for comparison purposes is to add the two delay terms in order to find the total delay due to the roundabout \(d\). This is shown in Equation 4-5.

\[ d = D + D_t \]

Equation 4-5: Average Facility and Queuing Delay
4.2 Data Collection

Data for testing the two-lane roundabout formulation was obtained from signalized intersections in suburban locations throughout the Twin Cities Metropolitan Area in Minnesota. The data collection sites are at the intersections of four-lane highways. The data that was collected and will be studied are turning movements during the afternoon peak hour.

A planning/engineering consultant collected this traffic data for use in traffic signal timing and intersection/interchange reconstruction projects. It was provided to the author for the purpose of testing in the context of this thesis.

Data for the comparison of this formulation to aaSIDRA was obtained from Dr. Rahmi Akcelik.

4.3 Sensitivity Testing

The first test will determine whether the formulation gives positive, non-infinite numbers, and appears to provide realistic results. The data for this test was taken from the 4:00 to 5:00 p.m. hour at the intersection of Lexington Avenue and Yankee Doodle Road in Eagan, Minnesota.

<table>
<thead>
<tr>
<th></th>
<th>NB</th>
<th>SB</th>
<th>EB</th>
<th>WB</th>
</tr>
</thead>
<tbody>
<tr>
<td>Left</td>
<td>170</td>
<td>127</td>
<td>139</td>
<td>64</td>
</tr>
<tr>
<td>Thru</td>
<td>206</td>
<td>612</td>
<td>598</td>
<td>594</td>
</tr>
<tr>
<td>Right</td>
<td>40</td>
<td>186</td>
<td>277</td>
<td>133</td>
</tr>
<tr>
<td>U-turn</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Total</td>
<td>416</td>
<td>925</td>
<td>1014</td>
<td>791</td>
</tr>
</tbody>
</table>
The turning movements used for this test are shown in Figure 4-2. This intersection served 2146 vehicles during this hour. These turning movements were entered into the formulation with all of the default values for the parameters, which include:

- Outside split = 0.8 (80% of circulating vehicles occupy the outside lane)
- Node-to-node distance = 120 feet (150 foot inscribed circle diameter)
- Circulating speed = 20 miles per hour
- Heavy vehicle percentage = 0
- Driver familiarity = 1.00 (no factor)
- Critical gap = 3.5 seconds (right lane), 4.5 seconds (left lane)
- Follow up time = 3 seconds (right lane), 3.5 seconds (left lane)

The results for this intersection, as calculated by the formulation are shown in Figure 4-3. A graphical illustration is shown in Figure 4-4. In the table, $q_a$ represents the entering volume, $Q_e$ represents the circulating volume at the entry, YLD the yield-line delay, TVD the traversal delay, and DEL represents the total average delay for the given approach.
Observation of the delays shows consistencies with expected results. The two approaches with the highest combined volumes (southbound and eastbound) have the greatest average yield line delays. Also, the approach with the greatest proportion of left turning vehicles (northbound) has the greatest average traversal delay.
Figure 4-5: Lexington and Yankee Doodle Delays

The next group of tests is intended to show monotonicity, robustness and continuity within the functions and parameters of the formulation. The first test used extreme volumes and volume ratios in an attempt to return unacceptable delays (negative or infinite) or errors. No matter what volumes were entered, the formulation always returned an acceptable delay. This is a result of the adjustment made based on Kimber’s (1989) principal of reverse priority. He stated that regardless of the density of the circulating roadway at the approach, entering vehicles would sooner or later create their own gap. This idea led to the minimum capacity of 33 pce per hour, which was stated as the minimum
capacity for two-way stop controlled intersections in the 1985 HCM. This test showed the robustness of the formulation; that it can handle any input.

The following tests were performed in order to demonstrate monotonicity, consistency and differentiability of the formulation output. These tests used turning movement data from 4:00 to 5:00 p.m. at the intersection of Trunk Highway 36 and Osgood Avenue in Stillwater, Minnesota. The results are based on the yield line, traversal, and total delays for the northbound vehicles. Figure 4-6 is a graphical illustration of how these delays are a function of the entering volume. For Figure 4-6, only the northbound entering volume was changed.
Figure 4-6 shows consistent relationships between entry volume and the resulting delays. The graph also shows monotonicity and differentiability.

For the next test the same data was used, and the entering volume at all other approaches was increased or decreased at the same rate. In this case, the northbound entering volume remained unchanged. Figure 4-7 shows the results of this test.

Figure 4-7 shows continuity, monotonicity and differentiability for the formulation. This graph also indicates that changes in the circulating volume has much more effect on the traversal delay, compared to the yield-line delay.
The final graph of this group shows the yield line, traversal, and average total delay when all of the entering volumes changed at the same rate. This includes every turning movement at every entry with the exception of U-turns. Figure 4-8 shows the effect that changing all the volumes has on delay.

Figure 4-8 shows continued monotonicity, continuity and differentiability of the formulation output. This graph combines the results of figures 4-6 and 4-7.

The preceding three graphs show that yield line delay is much more dependant on entering flow, while traversal delay is dependant on the circulating flow, and
the volume to capacity ratio of the circulating roadway. Figure 4-5 showed that circulating capacity had a much smaller effect on yield line delay, but this is also attributed to the northbound entry flow being far under capacity.

These graphs also show the monotonicity, continuity and differentiability of the formulation. There is consistency in the rise and run of the graphs. Had the graphs not shown any one of these characteristics, they would have uncovered an error in the formulation. Figure 4-9 is an example of these deficiencies.

![Figure 4-9: Hypothetical Examples of Errors in the Formulation](image)

The final sensitivity test investigates the effect geometry has on the formulation output. This test kept the volumes constant, while changing the circulatory speed and distance between each approach/exit leg. For this test, those distances were kept equal to each other, while being adjusted. The results of this test are shown in Figure 4-10.
4.4 Formulation Comparison

The following tests compare results from this formulation with those of aaSIDRA. Much of the formulation presented in this thesis is derived from the same sources as aaSIDRA. The volumes used for aaSIDRA were put into this formulation with certain assumptions being made about geometry and the parameters used.

The first comparison deals with a single lane roundabout with an inscribed diameter of 105 feet. That gives a distance of 83 feet between each approach at an assumed speed of 17 miles per hour. The values for distance and speed are
not definitely known. The distances are based on a perpendicular intersection, while the speed is normal for a one-lane roundabout. In order for the formulation to evaluate a single lane roundabout, several adjustments had to be made. The outside split and entering split parameters needed to be set to 1.0, and the volume portion of the traversal delay formula needed to be set to 1900. The turning movements also had to be converted for right-side driving, as aaSIDRA is an Australian program. Once this had been done, the volumes were inputted and the results compared.

Figure 4-11 shows the aaSIDRA results alongside the results from this formulation. The results are quite close, within 10%. In addition, as illustrated by the graph, the delays for each approach were roughly proportional between the two formulations.
The second comparison dealt with a two-lane roundabout. This comparison was difficult to make because of the need to make assumptions about geometry and slip lanes. From the aaSIDRA data it was not possible to determine what percentage of right turns used slip lanes, and also the distance between legs was unknown. After making the best assumptions, and adjusting the vehicle flows for right-side driving, results from both formulations were compared.

Figure 4-12 shows the results of this comparison. With the exception of one extremely high delay reported by aaSIDRA, the results are somewhat similar.
One difference that has been discovered from this comparison is that the spreadsheet for my formulation does not consider the traversal delay for vehicles using slip lanes. When implemented into a travel forecasting model, the network will compute this traversal time. Other reasons why the results from these two formulations were not more similar can be attributed to the driver characteristics of Australian drivers compared to American drivers. Geometrical differences and unknown roundabout speed also contributes to the difference in results. However, the proportional resemblance of the delays from aaSIDRA and my formulation are encouraging.
4.5 Summary

The inclusion of the traversal delay equation is important and necessary for testing and comparison. However, it is not required for implementation into a travel forecasting model. The travel forecasting model computes travel time on links as part of the simulation.

The preceding tests demonstrate that the formulation presented in this thesis functions correctly and resembles another proven simulation. The sensitivity tests demonstrate that this formulation provides realistic results that are consistent with changes in variables and parameters. In addition, tests show that even with unrealistic input volumes, positive, non-infinite, useable numbers will be produced. This is especially important for travel forecasting simulations that are based on all-or-nothing assignment, where very large flows can be assigned to links before capacity is considered.

The comparison of the produced formulation with aaSIDRA’s formulation showed similar patterns for delay. With some unknowns for geometry of the roundabout, the comparison still produced favorable results for the performance of the produced formulation. In a travel forecasting model, which operates at a much larger scale, the precision may not be as imperative as for application of aaSIDRA to a single, isolated roundabout.
5 Findings and Future Work

This chapter presents conclusions of this research and suggestions for future study of this topic. The findings portion of this chapter discusses the results of the literature review and the determination and selection of elements of the formulation. The last portion of this chapter focuses on recommendations for future research of the operation of and driver characteristics in multilane roundabouts in the United States.

5.1 Findings

The purpose of this thesis was to develop a formulation that can be implemented within a travel forecasting model. My formulation evaluates capacity and delay of multilane roundabouts in the United States. This objective was accomplished by reviewing literature and other research, the majority from northern Europe, Australia and New Zealand, and determining the most appropriate methods for U.S. applications.

The literature that was reviewed for this thesis had many topics:

- Roundabout history and operation
- Driver characteristics at roundabouts
- Gap acceptance principals
- Roundabout performance evaluation theory
Current U.S. travel forecasting models and traffic characteristics were used to select the most appropriate methods from those presented in the literature. Several questions were posed as an outline for the development of this formulation. These questions were presented in three groups.

The first group of questions was posed to determine the appropriate basis for the capacity/delay formulation. The second group dealt with the method of how the approaches would be evaluated. The third group of questions investigated the properties and integrity of the formulation. These questions will be addressed in the following sections.

5.1.1 Basis of the formulation

The selected basis of my formulation is a gap acceptance theory. There are two methods that are used to evaluate the performance of roundabouts, gap acceptance and empirical regression. Because of the framework of most existing travel forecasting models and the flexibility and interchangeability of gap acceptance based formulations, this theory was selected.
Regression models, based primarily on the geometrics of a roundabout, require intricate calibration and data from the specific intersection. In addition, it has been found that regression models are not reliable when applied to different facilities in different locations.

5.1.2 Level of Analysis of Approach Lanes

It was determined that analyzing the approach lanes on a lane-by-lane basis was most appropriate for my formulation. Other methods for determining the utilization of different entry lanes would be inaccurate or difficult to use a travel forecasting model.

The lane-by-lane analysis had previously worked well in gap acceptance-based models. Also, it was the most appropriate level of analysis for a network topology that treated each entry of the roundabout as a separate node and as a series of links for the circulating roadway.

My formulation bases the lane utilization on the principle of delay equilibrium. So although there is, in most cases, a different volume for each entry lane, there is one common value of delay for a given approach. This determination of lane
utilization is most appropriate for travel forecasting models where chosen paths are determined by minimizing disutility (travel time).

5.1.3 Quality and Integrity of the Formulation

Because my formulation is intended for implementation into large travel forecasting models, the required user input and complexity had to be kept to a minimum. Certain aspects of capacity and delay equations had to be eliminated or predetermined in order to lessen the work required for a user designing the network.

Based on the literature review and testing of the formulation with real data, it is apparent that my formulation will be useable for implementation in travel forecasting models. Certain variables were turned into constant parameters in order to maintain a low complexity to the formulation. These variables had a small enough effect so that the formulation was not too sensitive to the change.

The network topology that was selected for use with this formulation allows for great flexibility when designed and implemented in a model. It does not seem that any of the eliminated or adjusted variables significantly affect the integrity of this formulation. Also, the developed formulation produced similar results as aaSIDRA for a single lane and multi lane application. Although there were
unknowns in the geometry, the tendencies and trends of the delays demonstrated a reliable quality of my formulation.

5.2 Future Work

There are several topics related to multilane roundabout evaluation that can be research further. My formulation is primarily based on the review of previous research, and how existing theories and methods would work for this application. There are several areas where there was not significant research, or where empirical research and calibration should be done.

5.2.1 Circulating Split

This thesis has presented a formulation that incorporates the varying circulatory split of the roundabout. The shortcoming is that there are not true data or means to determine how the circulating split really functions in the United States. Future research should be done to determine how vehicles truly travel in the circulatory roadway based on several factors including circulating volume, different traffic maneuvers, roundabout speed and geometry, and many others.

Related to circulating split, and a topic also suitable for future work is the behavior of entering vehicles at multilane roundabouts. Research can focus on
the interaction between the use of the different entering lanes and how the driver characteristics differ when finding a gap and entering the circulatory roadway.

5.2.2  Driver Familiarity Factor

My formulation includes a factor that adjusts capacity based on the comfort of motorists using the roundabout. Based on the concept that capacity increases with greater driver familiarity with the facility, this factor is determined by the formulation user. However, there is no available research on how much effect driver inexperience has on the capacity of a roundabout.

Research to evaluate the effect of driver comfort could be conducted at any roundabout, but may require polling of drivers or a license plate inventory. Polling each driver would be a substantial task, and it could be otherwise difficult to determine that unfamiliar drivers affected the capacity. A license plate inventory would simply determine what proportion of the motorists using the roundabout are habitual users. A percentage could be related to the delay or flow at a particular facility. Also, the research would most likely require roundabouts that have high flow rates. This research could monitor certain multilane roundabouts over a five to ten year period in order to observe how capacity and overall facility use changes as drivers become more comfortable with the facility.
5.2.3 Calibration and Comparison

Further research can be conducted to determine how this formulation compares to many other models. Since much of this formulation is based on the same theory as aaSIDRA, it compares well with that formulation. Comparisons to other models should be made in order to determine what adjustments and calibrations can be made to the formulation developed for this thesis.
6 Bibliography


Buckhurst Fish & Jacquemart Inc. “Capacities of Modern Roundabouts.” www.bfjplanning.com


City of Lincoln, Nebraska, Public Works Department. “33rd Street and Sheridan Boulevard Roundabout Project.” www.lincolnroundabout.com


Oregon State University. “Modern Roundabouts.” www.engr.orst.edu/~taekrtha/round.html


Wallwork, Michael J. “Modern Roundabouts.” For the Roundabout Design Workshop, Montpelier, VT. April 1996.

### 7 Appendix – Various Formulation Spreadsheet Pictures

<table>
<thead>
<tr>
<th>NB Split</th>
<th>0.830175</th>
<th>Outsd Split</th>
<th>0.8</th>
<th>SB Split</th>
<th>0.654705</th>
<th>EB Split</th>
<th>0.643033</th>
<th>WB Split</th>
<th>0.666389</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>VOLUME</strong></td>
<td><strong>Semi%</strong></td>
<td><strong>RV/Bus%</strong></td>
<td><strong>Familiarity</strong></td>
<td><strong>GEOMETRICAL PARAMETERS</strong></td>
<td><strong>DISTANCE FROM NODE TO NODE</strong></td>
<td><strong>Circ Speed</strong></td>
<td><strong>25 mph</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LEFT</td>
<td>173</td>
<td>1</td>
<td>2</td>
<td><strong>NB-WB</strong></td>
<td>120 ft</td>
<td><strong>WB-SB</strong></td>
<td>120 ft</td>
<td><strong>SB-EB</strong></td>
<td>120 ft</td>
</tr>
<tr>
<td>THROUGH</td>
<td>206</td>
<td>1</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RIGHT</td>
<td>40</td>
<td>1</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>U-TURN</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>SB</strong></td>
<td>127</td>
<td>1</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>THROUGH</td>
<td>612</td>
<td>1</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RIGHT</td>
<td>186</td>
<td>1</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>U-TURN</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>EB</strong></td>
<td>139</td>
<td>1</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>THROUGH</td>
<td>277</td>
<td>1</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RIGHT</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>U-TURN</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>WB</strong></td>
<td>133</td>
<td>1</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>THROUGH</td>
<td>594</td>
<td>1</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RIGHT</td>
<td>133</td>
<td>1</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>U-TURN</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>3146</th>
<th><strong>Right Lane</strong></th>
<th><strong>Ent. Flow</strong></th>
<th><strong>Cir. Flow</strong></th>
<th><strong>Left Lane</strong></th>
<th><strong>Ent. Flow</strong></th>
<th><strong>Cir. Flow</strong></th>
<th><strong>Total Flow</strong></th>
<th><strong>Cir. Flow</strong></th>
<th><strong>Delay/pce</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>LEFT</td>
<td>173</td>
<td>352</td>
<td>706</td>
<td>72</td>
<td>882</td>
<td>424</td>
<td>882</td>
<td>10.01</td>
<td></td>
</tr>
<tr>
<td>THROUGH</td>
<td>213</td>
<td>618</td>
<td>677</td>
<td>326</td>
<td>846</td>
<td>944</td>
<td>846</td>
<td>26.74</td>
<td></td>
</tr>
<tr>
<td>RIGHT</td>
<td>181</td>
<td>665</td>
<td>656</td>
<td>369</td>
<td>820</td>
<td>1034</td>
<td>820</td>
<td>32.99</td>
<td></td>
</tr>
<tr>
<td>U-TURN</td>
<td>0</td>
<td>552</td>
<td>421</td>
<td>269</td>
<td>526</td>
<td>807</td>
<td>526</td>
<td>10.39</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Tot Del</th>
<th>Volume</th>
<th>Delay/pce</th>
</tr>
</thead>
<tbody>
<tr>
<td>NB-WB</td>
<td>173</td>
<td>4858</td>
</tr>
<tr>
<td>WB-SB</td>
<td>130</td>
<td>11996</td>
</tr>
<tr>
<td>SB-EB</td>
<td>0</td>
<td>11331</td>
</tr>
<tr>
<td>EB-NB</td>
<td></td>
<td>8571</td>
</tr>
</tbody>
</table>

Adjustment for one lane (c=1900 if 1, c=2375 if 2)
Northbound

<table>
<thead>
<tr>
<th>Right Lane</th>
<th>Left Lane</th>
</tr>
</thead>
<tbody>
<tr>
<td>beta = 3 sec/veh</td>
<td>beta = 3.5 sec/veh</td>
</tr>
<tr>
<td>Delta C = 1.2 sec/veh</td>
<td>Delta C = 1.2 sec/veh</td>
</tr>
<tr>
<td>q_c = 706 pcu/h</td>
<td>q_c = 882 pcu/h</td>
</tr>
<tr>
<td>p Unbunched in circulating stream = 0.55</td>
<td>Unbunched = 0.55 1 is random</td>
</tr>
<tr>
<td>lambda = 706</td>
<td>lambda = 882</td>
</tr>
<tr>
<td>alpha = 3.5 sec/veh</td>
<td>alpha = 4.5 sec/veh</td>
</tr>
<tr>
<td>Q_e (Capacity) = 708 veh</td>
<td>Q_e = 431</td>
</tr>
</tbody>
</table>

Southbound

<table>
<thead>
<tr>
<th>Right Lane</th>
<th>Left Lane</th>
</tr>
</thead>
<tbody>
<tr>
<td>beta = 3 sec/veh</td>
<td>beta = 3.5 sec/veh</td>
</tr>
<tr>
<td>Delta C = 1.2 sec/veh</td>
<td>Delta C = 1.2 sec/veh</td>
</tr>
<tr>
<td>q_c = 677 pcu/h</td>
<td>q_c = 846 pcu/h</td>
</tr>
<tr>
<td>p Unbunched in circulating stream = 0.55</td>
<td>Unbunched = 0.55 1 is random</td>
</tr>
<tr>
<td>lambda = 677</td>
<td>lambda = 846</td>
</tr>
<tr>
<td>alpha = 3.5 sec/veh</td>
<td>alpha = 4.5 sec/veh</td>
</tr>
<tr>
<td>Q_e (Capacity) = 724 veh</td>
<td>Q_e = 447</td>
</tr>
</tbody>
</table>

Northbound

<table>
<thead>
<tr>
<th>Right Lane</th>
<th>Left Lane</th>
</tr>
</thead>
<tbody>
<tr>
<td>dm = 5.083686 seconds dm = 3600/Q_e</td>
<td>dm = 8.343546 seconds dm = 3600/Q_e</td>
</tr>
<tr>
<td>T_f = 0.25 hours T_f = 0.25 hours</td>
<td>T_f = 0.25 hours T_f = 0.25 hours</td>
</tr>
<tr>
<td>z = -0.502544 Performance parameter</td>
<td>z = -0.832989 Performance parameter</td>
</tr>
<tr>
<td>K_d = 1 Overflow parameter</td>
<td>K_d = 1 Overflow parameter</td>
</tr>
<tr>
<td>x = 0.497456 Degree of saturation = q_a/Q_e</td>
<td>x = 0.167011 Degree of saturation = q_a/Q_e</td>
</tr>
<tr>
<td>Q_e = 708 Veh/h Q_e = 431 Veh/h</td>
<td>Q_e = 431 Veh/h Q_e = 72 Veh/h</td>
</tr>
<tr>
<td>N_i = 0 vehicles N_i = 0 vehicles</td>
<td>N_i = 0 vehicles N_i = 0 vehicles</td>
</tr>
<tr>
<td>Q_a = 352 Veh/h Q_a = 72 Veh/h</td>
<td>Q_a = 72 Veh/h Q_a = 326 Veh/h</td>
</tr>
<tr>
<td>Delay = 10.01</td>
<td>Delay = 10.01</td>
</tr>
<tr>
<td>Delay Difference = 0.00</td>
<td></td>
</tr>
</tbody>
</table>

Southbound

<table>
<thead>
<tr>
<th>Right Lane</th>
<th>Left Lane</th>
</tr>
</thead>
<tbody>
<tr>
<td>dm = 4.972917 seconds dm = 3600/Q_e</td>
<td>dm = 8.046679 seconds dm = 3600/Q_e</td>
</tr>
<tr>
<td>T_f = 0.25 hours T_f = 0.25 hours</td>
<td>T_f = 0.25 hours T_f = 0.25 hours</td>
</tr>
<tr>
<td>z = -0.146632 Performance parameter</td>
<td>z = -0.271934 Performance parameter</td>
</tr>
<tr>
<td>K_d = 1 Overflow parameter</td>
<td>K_d = 1 Overflow parameter</td>
</tr>
<tr>
<td>x = 0.853368 Degree of saturation = q_a/Q_e</td>
<td>x = 0.728066 Degree of saturation = q_a/Q_e</td>
</tr>
<tr>
<td>Q_e = 724 Veh/h Q_e = 447 Veh/h</td>
<td>Q_e = 447 Veh/h Q_e = 72 Veh/h</td>
</tr>
<tr>
<td>N_i = 0 vehicles N_i = 0 vehicles</td>
<td>N_i = 0 vehicles N_i = 0 vehicles</td>
</tr>
<tr>
<td>Q_a = 618 Veh/h Q_a = 326 Veh/h</td>
<td>Q_a = 326 Veh/h Q_a = 72 Veh/h</td>
</tr>
<tr>
<td>Delay = 26.74</td>
<td>Delay = 26.74</td>
</tr>
<tr>
<td>Delay Difference = 0.00</td>
<td></td>
</tr>
</tbody>
</table>